Soft Mud Properties: Voigt Model

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Introduction

Significant wave damping has been reported on muddy coasts where the beds are composed of soft movable muds (Tubman and Suhayda 1976; Wells 1983). This phenomenon has also been observed in the laboratory (Schuckman and Yamamoto 1982; Maa 1986). The most likely reason for this great wave attenuation is related to mud motion, and therefore, knowledge concerning the dynamic properties of soft muds is needed eagerly.

Experiments on the response of typical low water-content clayey soils under cyclic shear loadings have been reported. The response of shear strain versus shear stress can be described by a hysteresis loop (Hardin and Drnevich 1972). The energy dissipation is defined as the area enclosed by the loop. The ratio of energy dissipation is defined as the ratio of the energy loss and the elastic energy stored in the specimen when the strain is a maximum (Kolsky 1963). For a soil specimen experiencing a large cyclic deformation, the enclosed area would be increased because of the nonlinear response. Kovacs et al. (1971) showed that the ratio of energy dissipation increases as the shear strain increases. To simulate the described behavior, Thiers and Seed (1968) suggested the bilinear model. This approach might cause difficulty in handling the equation of motions. Here a rather simple viscoelastic model (Malvern 1969) is introduced and it can be easily applied to solve for the motion of muds (Maa 1986).

The soil characteristics described in the last paragraph is for clays with low water content or well-consolidated clays. For those clayey muds in estuaries, bays, or coasts, a typical feature is the high water content or low shear strength. Maa (1986) reviewed five possible constitutive models for soft muds. He also compared the motion of mud beds predicted by several models. He concluded that the two simple viscoelastic models, the Voigt model and the Maxwell model (Malverne 1969), are reasonable choices. Albeit the Voigt model has been selected by MacPherson (1980) and Hsiao and Shemdin (1980), they did not explain why the Voigt element is a better selection than the Maxwell model. The objective is to demonstrate that the Voigt element is indeed a better choice compare with the Maxwell model. Correlation among the three material constants, shear modulus, viscosity, and density, are also presented. It is not to say that the Voigt model is completely satisfied in representing the rather complex behavior of muds. However, the goal is to find a relatively reasonable predictive model.

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Simple Viscoelastic Models

Viscoelastic models account for the energy loss in an indirect manner. It is assumed that the restoring forces are proportional to the amplitude of vibration, while the dissipative forces are proportional to the velocity.

The two simple linear models, the Voigt and the Maxwell elements, are based on the analogy of a spring-dashpot system, see Fig. 1. The relevant constitutive equations are

Voigt element: $$T' = 2GE' + 2\mu\dot{E}' \quad (1a)$$

Maxwell element: $$T' + \left(\frac{\mu}{G}\right)\dot{T}' = 2\mu\dot{E}' \quad (1b)$$

where G = the shear modulus of elasticity; μ = viscosity of mud; T and E = the stress and strain tensors, respectively; the prime indicates the deviator part of those two tensors; and the dot indicates the derivative with respect to time. Fig. 1 also shows the shear strain, E', as a function of time under a constant shear load. For the Voigt element, the response of elastic shear strain is delayed, but it finally reaches the value, $T'/2G$, that the spring alone (representing purely elastic response) would reach. However, the Maxwell element will keep creeping because of the presence of the dashpot.

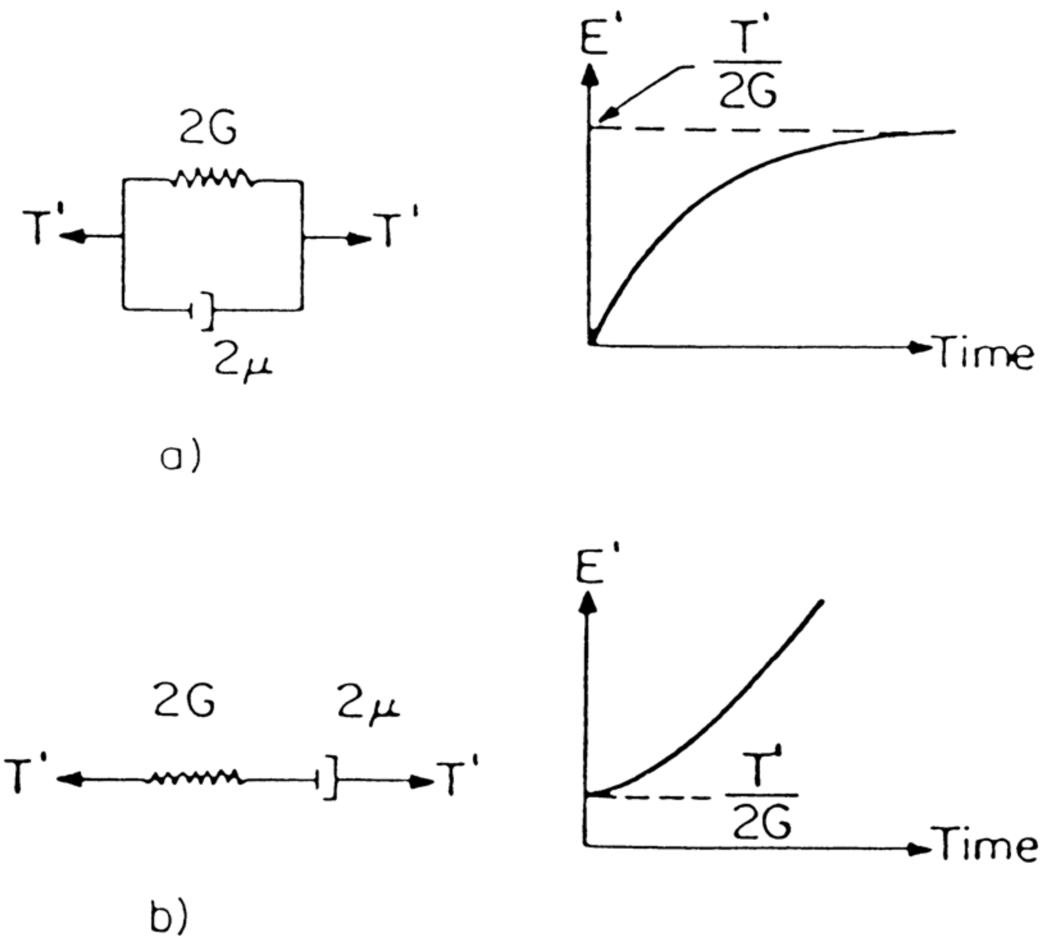

FIG. 1. System Diagram for Two Simple Viscoelastic Models and Response under Constant Loading: (*a*) Voigt Element; and (*b*) Maxwell Element

Relaxation Tests

As pointed out previously, the Maxwell element exhibits creep at a constant shear load but the Voigt element stops at a finite deformation. Therefore, a constant shear load on the mud and the resultant angular displacement information can be utilized as a means to check whether the mud behavior is closer to the Voigt or the Maxwell element. A Brookfield viscometer, a miniature vane, an angular displacement pointer, and a video camera were assembled to perform these tests. Details are given elsewhere (Maa 1986). Six tests (runs 1–6) are performed. The first three were beds of kaolinite, and the remaining three were beds of a mixture of kaolinite and natural estuarial mud from Cedar Key, Florida. The vane, which was inserted in the mud at a predetermined elevation, was driving by the motor via a calibrated spiral spring. The applied torque and corresponded angular displacement were recorded from the dial meter and the angular displacement pointer via a movie camera. The response at the first few seconds are used to determine the material constant, G and μ (to be discussed later). The system ran for a sufficiently long time and reached an equilibrium state, while the motor and the van rotated at the constant speeds. The motor was then turned off, and the response of angular displacement and residual torque were again recorded via a video camera. These responses were used to judge if the tested muds could resist any shear stress. Albeit the muds are disturbed, the results will lead to a more conservative conclusion.

Results

Fig. 2 shows the time history of the residual torque (which corresponds to the shear stress) and angular displacement of the vane for run 1, consolidated for a period of seven days. Negative numbers, e.g., −1.5 cm, indicate the depth from the mud surface to the center of the vane. A sharp decrease in the torque, along with a rapid increase in the angular displacement, was

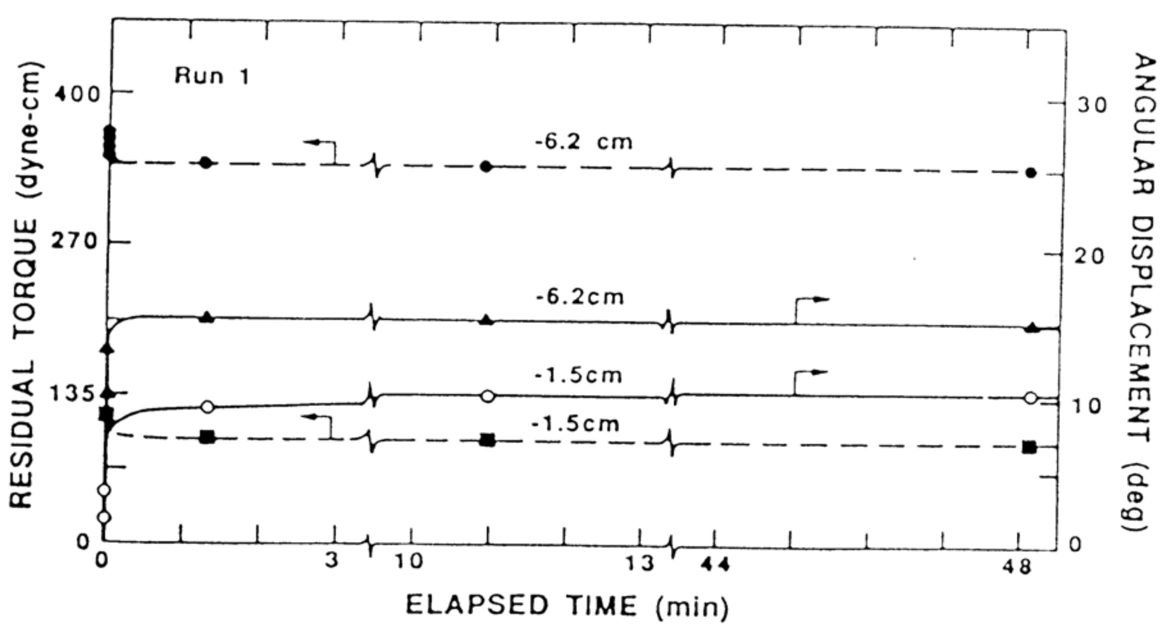

FIG. 2. Plot of Residual Torque and Angular Displacement versus Elapsed Time for One-Week Consolidated Kaolinite Bed

observed during the first few seconds. Later the torque and angular displacement changed much less significantly. After about 5 min, the torque and the angular displacement pointer did not change any further, and an equilibrium state was attained. The finite residual torque indicates non-relaxation of the shear stress in the mud under a stationary shear load. The rapid response at the first few seconds after turning off the motor was due to the large inertia of the vane (made by stainless steel) but with little contribution from mud creep. The described experiments thus provided evidence that the selected muds did not behave as a Maxwell element since they resisted a constant shear stress. Material constants for the Voigt element, G and μ, were next obtained as follows.

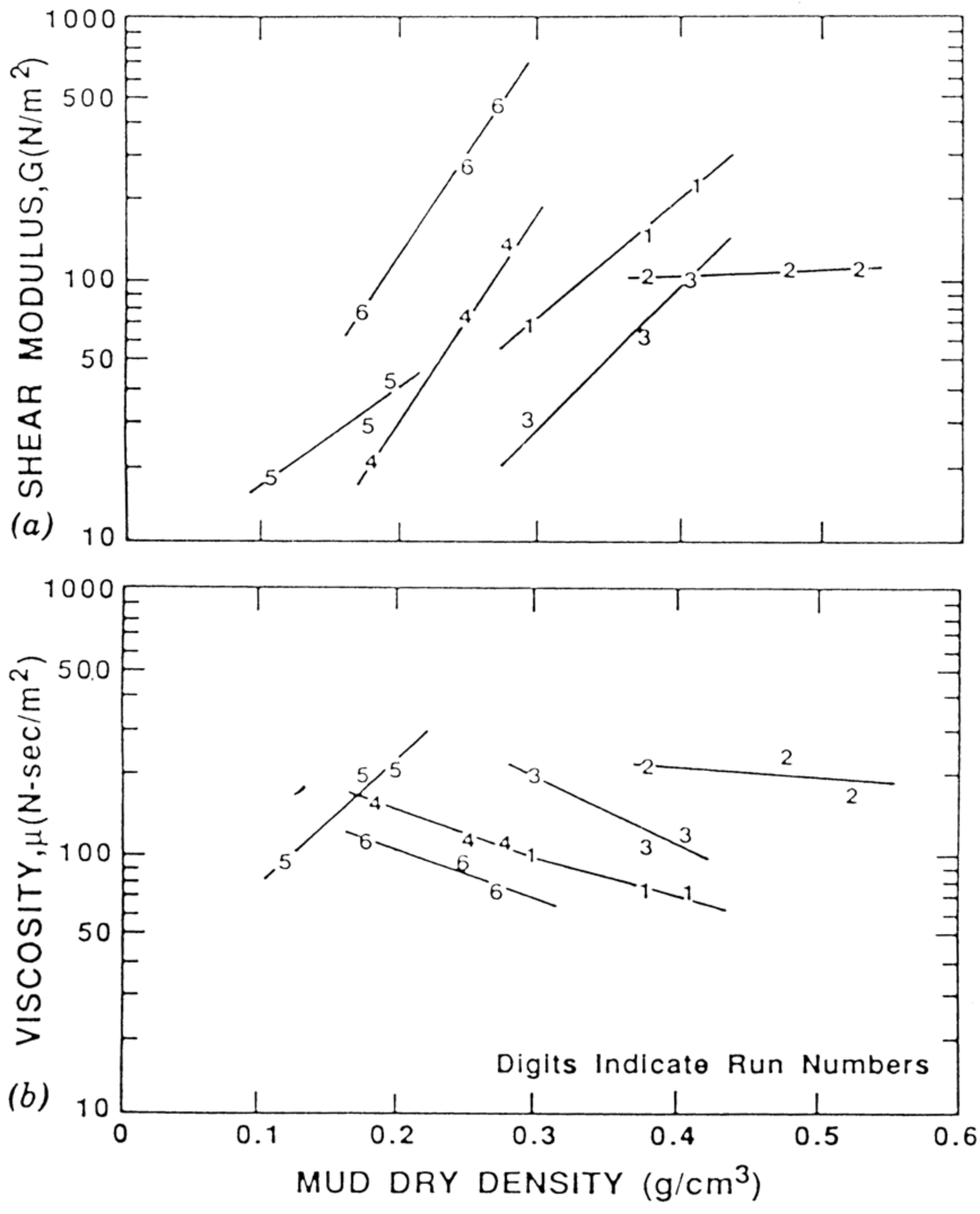

FIG. 3. Relationship between Viscoelastic Constants and Mud Dry Density: (*a*) Viscosity; and (*b*) Shear Modulus

Measurements of Material Constants

The initial stress-strain behavior, as applied a shear loading to the mud bed, was used to obtain the material constants. The maximum shear strain considered in this study was less than 5° (0.087 rad), which can be considered to be a small strain. The Voigt model is applicable for this case of a reasonably linear relationship between shear stress and shear deformation. The slope of the representative straight line for shear stress and strain gives the shear modulus. The intersection of this straight line at the shear stress axis gives the viscosity.

The vane was run at three elevations to give information on the vertical variation of shear modulus and viscosity. Vertical profiles of mud density were also measured (Maa 1986). Figs. 3(*a*) and 3(*b*) show the relationship between μ and G versus mud density.

The shear modulus appears to be well correlated with the density except for the fully consolidated mud (run 2, consolidated for 14 days); see Fig. 3(*a*). The correlation of viscosity to dry density (see Fig. 3[*b*]) indicates that the viscosity generally decreases as the density increases. This situation reverses if the density is less than a specific value, e.g., 0.2 g/cm^3; see run 5. This may imply a critical value corresponding to a different behavior, noting that the viscosity must eventually decrease to that of water as the dry density approaches zero. When viewed in the light of the soil behavior described in the introduction, this behavior would be expected since soft muds exhibit higher shear strain and, consequently, more energy dissipation, i.e., higher damping, than denser muds. However, further considerations on the observed trends must await more data.

Concluding Remarks

The Voigt viscoelastic model is indeed a better selection for modeling soft mud bed responses because tested muds did not exhibit the creep phenomenon characteristic of the Maxwell model, but rather showed the ability to resist a shear force and a reasonably linear response for small strain. In the Voigt model, μ serves as an index of internal friction loss. It has a somewhat different physical meaning than the viscosity of a Newtonian fluid. Therefore, a dense mud has a lower value of μ than a soft one because of the lesser mobility.

Acknowledgment

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