MS698–3: Sediment transport processes in coastal environments
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Lecture 10: Erosion and Deposition

- Transport equation.
- Deposition and erosion terms.
- Erosion equation.
- Flux convergence and divergence.

Class business

- This **Friday, 11:00**: Elgar ht fl. paper; classroom C

Materials used

- Nielsen (1992), Section 5.3.

Today we look at two ways to estimate net erosion and deposition. One invokes a relationship between local bed shear stress, sediment characteristics, and suspended sediment concentrations. The other invokes a depth-integrated statement of conservation of sediment mass.

Transport Equation

For non-steady, non-uniform suspended transport, conservation of sediment mass applied to an arbitrary control volume is stated as the *Transport equation* or *Advection-Diffusion equation*:

$$\frac{\partial c_s}{\partial t} = -\frac{\partial (u c_s)}{\partial x} + \frac{\partial}{\partial x}\left(K_x \frac{\partial c_s}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial c_s}{\partial z}\right) + w_s \frac{\partial c_s}{\partial z}; \qquad (1)$$

where K_x is horizontal diffusivity, K_z is the vertical eddy diffusivity (discussed last week), and w_s is sediment settling velocity. The horizontal velocity (u), and suspended sediment concentration represent Reynolds-averaged values. This equation assumes that vertical flow velocities (w) are very low, and that the flow is two-dimensional ($\partial/\partial y$ low). To get to equation 1 you simply Reynold's average the conservation of mass equation from last week's lecture, without making the assumptions of steady and uniform conditions. The *advective* terms in equation 1 are the velocity terms, while the *diffusive* terms are the K_x and K_z terms.

Assuming uniform conditions, the vertical balance becomes

$$\frac{\partial c_s}{\partial t} = \frac{\partial}{\partial z}\left(K_z \frac{\partial c_s}{\partial z}\right) + w_s \frac{\partial c_s}{\partial z}; \qquad (2)$$

$$\frac{\partial c_s}{\partial t} = \frac{\partial}{\partial z}\left(K_z \frac{\partial c_s}{\partial z} + w_s c_s\right).$$

Last week, we looked at the steady version of this equation, and derived solutions that used a reference concentration c_a as a bottom boundary condition. Suppose, however, that the flow and sediment mixture are not at steady state, but are changing with time. It would not be appropriate to assume that the reference concentration is a valid means for setting the bottom 2boundary condition. Instead, a *flux boundary condition* should be used, that specifies how much sediment @is being exchanged between the bed and the water column. Following Parker (1978), who shows why flux boundary conditions are a good idea, we note that the net erosion or deposition from the bed is a balance between sediment that settles to the bed ($w_s c_a = D$) and sediment that is diffused away from the bed ($E = e w_s$), where D and E are deposition and erosion, and e is an erosion coefficient. The net leaving or entering the bed is the difference between D and E, and must equal the difference between settling and diffusion.

$$E - D = (e - c_a)\, w_s = c_a w_s + K_z \frac{\partial c_s}{\partial z} \|_{z=z_a}. \qquad (3)$$

The erosion coefficient, e, is usually assumed to be related to the excess shear stress of the sediment, and to sediment availability. The boundary condition is applied to equations 1 and 2 as $\partial c_s/\partial z = e/K_z(z_a)$. Once the suspension has reached a steady state, erosion and deposition balance each other. This occurs when the concentration of sediment near the bed (c_a) exactly equals the erosion coefficient of the flow e. When the concentration is higher ($c_s > e$), net deposition occurs; when it is lower ($c_a < e$), net erosion occurs.

Entrainment rates

Many formulations for e can be found in the literature, usually segregated into relations for sandy material (see Nielsen, 1992) and for cohesive material. Garcia and Parker (1991) review a number of commonly used bottom boundary conditions, an d Sanford and Maa (2001) revisit these. Most common for cohesive sediments include the Partheniades (1965) formulation

$$E = \frac{dm}{dt} = M\left(\frac{\tau_b - \tau_{cr}}{\tau_{cr}}\right);\ for\ \tau_b > \tau_{cr}; \qquad (4)$$

where M (=0.00001 − 0.0005 $kg/(m^2 s)$) is a coefficient that depends on sediment characteristics, and m is the mass of sediment on the bed. E is erosion rate (mass) per unit area of the bed. For sandier material, van Rijn (1984) found that the sediment entrainment rate (or "pickup function") increases with $S^{3/2}$.

I used a value of e that would reach a Smith and McLean type steady-state concentration in Harris and Wiberg (2002, 2001), $e = c_b f \gamma_0 S / (1 + \gamma_0 S)$ where f is bed fraction, γ_0 is the suspension coefficient, and $S = (\tau_b - \tau_{cr}) / \tau_{cr}$ is excess shear stress.

Deposition rates

The deposition term is treated very simply in equation 3 as being a constant settling term. Many researchers do not apply this for deposition of fine-grained silts and clays. They instead assume a critical threshold for deposition $\tau_{cr,d}$; above which the sediment will not deposit. Another approach is to reduce the depositional settling term to account for effects such as hindered settling. Other researchers include a probability of sticking $0 < P < 1$ that depends on shear stress so that $D = P w_s c_a$.

Flux convergence and divergence

The lectures on suspended sediment transport assumed that conditions governing flux were both steady and uniform. Under these conditions, the sediment bed is not modified by the transport. If transport is either time-dependent or spatially non-uniform, however, conservation of sediment mass dictates that there must be some source of or sink for sediment. The most likely external source or sink is from the sediment bed itself, i.e. through erosion or deposition. To explore conservation of sediment mass we need to define two depth integrated quantities; depth-integrated flux (q_s) and volume of sediment suspended (V_s).

$$V_s = \int_{z_a}^{h} c_s dz. \tag{5}$$

V_s represents the amount of sediment that is suspended within some water column, and has units of volume / area. If adjusted by porosity, V_s/c_b represents the equivalent depth of bed sediment accounted for by the suspended sediment. In many systems, V_s/c_b represents less than a centimeter of bed sediment. To get large values of net erosion or deposition, therefore, requires more than simply putting material into suspension. Total depth-integrated flux (q) would be the vector sum of bedload flux (q_{bx}, q_{by}) and suspended flux (q_{sx}, q_{sy}).

$$\begin{aligned} q_{sx} &= \int_{z_a}^{h} c_s u dz; \\ q_{sy} &= \int_{z_a}^{h} c_s v dz; \end{aligned} \tag{6}$$

where z_a is the base of the suspension layer, h is water depth, $c_s(z)$ is the suspended sediment concentration (volume / volume), u is the horizontal velocity in the x-direction, and v in the y-direction. Note that equations 5 and 6 can be rewritten as depth-integrated mass of suspended sediment and mass flux by including sediment density.

Consider a control volume where the sources of sediment are the bed and flux from upstream, and sinks are the bed and downstream transport. The mass of sediment into the control volume must equal the mass out ± the source / sink term. Here, the source and sink must be erosion and deposition from the bed.

Steady Flow

Consider a steady flow ($\partial/\partial t = 0$). If there are horizontal gradients in sediment flux, sediment from the bed must be either eroded or deposited. If more horizontal flux enters the volume than leaves it, deposition must have occurred; whereas if sediment leaves the control-column at a faster rate than it enters the column- erosion must have occurred. This can be summarized

$$\begin{aligned} Erosion: &\quad \tfrac{\partial \eta}{\partial t} < 0;\ \ (q)_{out} > (q)_{in}\,; \\ Deposition: &\quad \tfrac{\partial \eta}{\partial t} > 0;\ \ (q)_{out} < (q)_{in}\,; \end{aligned} \tag{7}$$

where η is the height of the sediment bed. As our control-column gets small; $\left[(q_x)_{out} - (q_y)_{in}\right]/\partial x = \frac{\partial q_x}{\partial x}$. Equation 7 can be rewritten as

$$\begin{aligned} \frac{\partial q_x}{\partial x} &\sim -\frac{\partial \eta}{\partial t}; \\ \frac{\partial q_y}{\partial y} &\sim -\frac{\partial \eta}{\partial t}. \end{aligned} \tag{8}$$

or, in two directions

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} \sim -\frac{\partial \eta}{\partial t}. \tag{9}$$

Horizontal gradients of depth-integrated flux represent flux convergence ($\partial q/\partial x < 0$) and flux divergence ($\partial q/\partial x > 0$). You can have transport without having either erosion or deposition in a steady system that has no flux convergence or divergence.

Uniform Flow

Besides flux convergence and divergence, sediment can be eroded or deposited when there are temporal variations in sediment concentration. Imagine a uniform flow ($\partial/\partial x = \partial/\partial y = 0$) at time t, with depth-integrated sediment suspension V_s. Sediment must have been supplied from the bed if more sediment is in suspension at time $t + \Delta t$, $V_s(t + \Delta t) > V_s(t)$. Likewise, if sediment concentrations decrease within the time-interval,$V_s(t + \Delta t) < V_s(t)$, then sediment must have been deposited to the bed. This is because the upstream flux is not supplying sediment any faster than it is carried away ($\partial/\partial x = 0$). For uniform flow, therefore

$$\begin{aligned} Erosion: \quad & \tfrac{\partial \eta}{\partial t} < 0;\ V_s\left(t+\Delta t\right) > V_s\left(t\right) \\ Deposition: \quad & \tfrac{\partial \eta}{\partial t} < 0;\ V_s\left(t+\Delta t\right) < V_s\left(t\right). \end{aligned} \tag{10}$$

Letting our time-interval get small we can derive

$$\frac{\partial V_s}{\partial t} \sim -\frac{\partial \eta}{\partial t}. \tag{11}$$

Erosion Equation

From equations 9 and 11 we see that both temporal and spatial gradients in sediment suspension contribute to erosion and deposition of the bed. Or it might be that erosion and deposition must contribute either temporal or spatial gradients to sediment flux. The ratio of erosion depth (or deposition depth) to the volume of sediment removed from (or added to) the bed per unit area is c_b; the concentration of bed sediment. The porosity of the bed equals $1 - c_b$. Combining equations 9 and 11, we derive the *Erosion Equation:*

$$-\frac{\partial \eta}{\partial t} = \frac{1}{c_b}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial V_s}{\partial t}\right]. \tag{12}$$

This is sometimes called the *Exner Equation*. It states conservation of sediment mass for the combined flow and bed system. It assumes a constant porosity for the sediment bed. If we use sediment mass in suspension, and sediment mass flux, the equation can be rewritten to represent the mass of sediment eroded or deposited, instead of an erosion/deposition depth.

$$-\frac{\partial M_b}{\partial t} = \rho_s\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial V_s}{\partial t}\right]; \tag{13}$$

where M_b is the mass of sediment on the bed, and ρ_s is the density of sediment ($\approx 2.65 g/cm^3$). To relate changes in the mass of bed to erosion depth requires that we know the bulk density of the sea bed.

Multiple grain sizes

Most often sediment beds include size distributions of sediment, and not a single grain size. To estimate erosion and deposition of each grain size, equations 3, 12, or 13 should be applied to each sediment size. In the case of a flux boundary condition (equation 3), the erosion rate is usually adjusted by multiplying it by the fraction that each individual sediment size makes up on the bed. The erosion equation (equations 12 and 13) are applied to each sediment size independently. This approach allows feedbacks between the sediment size distribution on the bed and the suspended sediment size distribution. It also can be used to account for modifications to seabed texture through erosion and deposition.

Local vs. Depth-integrated approach

We have considered two ways to estimate net erosion and deposition. One relies on a local specification of flow and sediment properties, the other on depth-integrated fluctuations in flux and suspended sediment concentration. The first requires that you can estimate with some accuracy shear stress at the bed, sediment bed properties (size and τ_{cr}), and settling properties near the bed. The depth-integrated approach assumes that you have confidence in your estimates of total suspended flux, and suspended sediment volume, as well as temporal and spatial gradients in these.

References

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