MS698–3: Sediment transport processes in coastal environments
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Lecture 8: Suspended Sediment Transport

- Threshold for suspension.
- Conservation of mass for suspended sediment.
- Eddy diffusivity.
- Rouse profile.
- γ_0

Class business

- This Thursday: Bentley, Keen, Blaine paper.

Handouts

- Schematic of suspended sediment transport; Rouse profiles.

Threshold for suspension

Suspension occurs under high shear stress flows, when turbulent fluctuations in the vertical velocity of the flow (w') are as strong (or stronger) than the settling velocity of the sediment ($\underline{w_s}$). (*Note: I'm underlining settling velocity to distinguish it from the vertical velocity of the sediment.*) When $w' \approx \underline{w_s}$ the sediment will be barely in suspension; this is called *incipient suspension.* The vertical velocity fluctuations scale with the friction velocity of the flow ($w' \approx u_* \times (0.8 - 1.0)$); so that the criteria for incipient suspension is $\underline{w_s} \approx u_*$. We can define the ratio of settling velocity to vertical fluctuations by the *Rouse parameter*, P:

$$P = \frac{\underline{w_s}}{\kappa u_*}; \tag{1}$$

where κ is von Karman's constant (0.408). The criteria for suspension can be written as:

$$\begin{aligned} P = \frac{\underline{w_s}}{\kappa u_*} &> 2.5;\ \textit{no suspension}; \\ 1 < P = \frac{\underline{w_s}}{\kappa u_*} &< 2.5;\ \textit{incipient suspension}; \\ P = \frac{\underline{w_s}}{\kappa u_*} &< 1;\ \textit{full suspension}. \end{aligned} \tag{2}$$

This can be rewritten in terms of a critical shear stress for suspension, $\tau_b > \rho \underline{w_s}^2$ for incipient suspension and $\tau_b > 6.25 \rho \underline{w_s}^2$ for full suspension. Comparing the suspension threshold to the critical shear stress of motion for a range of grain sizes, it is seen that, for fine-grained sediment ($D < 63 - 88 \mu m$) the suspension threshold is less restrictive than the transport threshold. Silts and clays, therefore, will be suspended as soon as they become mobile and will not tend to be transported as bedload. For sands, however, the transport threshold is less restrictive than the suspension threshold, and they will transport as both suspended and bedload.

Turbulence to diffuse sediment upward is required to suspend sediment. Once suspended, sediment will be mixed throughout the turbulent portion of the flow. This confines suspended transport to turbulent boundary layers.

General Definitions and Concepts

Define the *volumetric concentration* of sediment to be $c_s = V_s / V_T$; where c_s is suspended sediment concentration (volume of sediment per total volume). The *mass concentration*, C_s, is defined to be the mass of suspended sediment per total volume ($C_s = \rho_s c_s$), usually given in units of mg/L, g/L, or g/m^3. For this lecture we'll use the volumetric concentration of sediment.

The *suspended sediment flux*, q_{ss} is the horizontal transport rate of sediment,

$$q_{ss} = \int_0^h c_s u_s dz = \int_0^h c_s u dz; \tag{3}$$

where u_s is the sediment velocity. In the second half of equation **??**, we assume that the horizontal sediment velocity (u_s) equals fluid velocity (u). It is normal for the velocity to increase, and sediment concentration to decrease above the bed; but we need an expression for the sediment profile in order to estimate flux using equation **??**.

Conservation of Mass for Suspended Sediment

By conserving sediment mass within the water column, and making assumptions about the shape of the turbulent velocities, we can derive relationships for the vertical suspended sediment profile. A general statement of conservation of sediment mass is

$$\frac{\partial c_s}{\partial t} + \frac{\partial c_s u_s}{\partial x} + \frac{\partial c_s v_s}{\partial y} + \frac{\partial c_s w_s}{\partial z} = 0; \tag{4}$$

where u_s, v_s, and w_s are the components of sediment velocity. If we assume steady state ($\partial c_s / \partial t = 0$), and that the flow is uniform in the horizontal direction ($\partial / \partial x = \partial / \partial y = 0$), this simplifies to $\partial \left(c_s w_s \right) / \partial z = 0$. Next, remember that the flow is turbulent. We can treat the time-averaged and fluctuating components of sediment concentration and velocity separately using the definitions

$$\begin{aligned} c_s &= \overline{c_s} + {c_s}'; \\ w_s &= \overline{w_s} + {w_s}'; \end{aligned}$$

$$
\begin{aligned}
u &= \overline{u} + u'; \\
v &= \overline{v} + v'; \\
w &= \overline{w} + w'; \qquad (5)
\end{aligned}
$$

where the over-bar represents the time-averaged quantity, and the "prime" represents the turbulent fluctuation. The statement of conservation of suspended sediment mass for steady, uniform flow can be written

$$
\begin{aligned}
\frac{\partial}{\partial z}\left[\left(\overline{c_s} + {c_s}'\right)\left(\overline{w_s} + {w_s}'\right)\right] &= 0; \\
\frac{\partial}{\partial z}\left[\overline{c_s}\ \overline{w_s} + \overline{c_s}{w_s}' + {c_s}'\overline{w_s} + {c_s}'{w_s}'\right] &= 0. \qquad (6)
\end{aligned}
$$

Next, by taking the time-average of both sides of equation **??**, we can disregard two of the terms. Remember the definition of a time-average: $\overline{f(z)} = \frac{1}{T}\int_T f(z)dt$. Time-averaging equation **??** we get:

$$
\frac{\partial}{\partial z}\left[\overline{c_s}\ \overline{w_s} + \overline{{c_s}'{w_s}'}\right] = 0. \qquad (7)
$$

Next, we think about what the vertical sediment velocity should be ($w_s = \overline{w_s} + {w_s}'$). It ($w_s$) should equal the sum of the vertical fluid velocity ($w = \overline{w} + w'$) and the sediment settling velocity ($\underline{w_s}$). Because the flow is steady and uniform in the horizontal; the mean vertical flow velocity is zero ($\overline{w} = 0$), so that the total vertical sediment velocity is equal to $w_s = \overline{w_s} + {w_s}' = w' + \underline{w_s}$. Through time-averaging, we can conclude that the fluctuating part of the sediment velocity derives from fluctuations in turbulent velocities, and that the time-averaged part equals the settling velocity of the sediment. Equation **??** therefore becomes

$$
\underline{w_s}\frac{\partial \overline{c_s}}{\partial z} + \frac{\partial \overline{({c_s}'w')}}{\partial z} = 0; \qquad (8)
$$

assuming that the settling velocity is independent of height.

The final step is dealing with the $\overline{({c_s}'w')}$ term. This term represents vertical diffusion of suspended sediment by turbulence.

Eddy Diffusivity for Mass

Remember that in the equation for turbulent momentum balance, a similar term was equated to the product of the eddy viscosity and velocity gradients; $\overline{(u'w')} = K\frac{\partial u}{\partial z}$, where K is the eddy viscosity that parameterizes the mixing of momentum. A similar argument has been made for treating the mixing of sediment, in part because the turbulent eddies that mix momentum are the same as the turbulent eddies that mix sediment mass.

$$
\overline{({c_s}'w')} = K_s\frac{\partial c_s}{\partial z}; \qquad (9)
$$

where K_s is the *eddy diffusivity*, and has units of $length^2/mass$. Equation **??** becomes

$$\frac{\partial}{\partial z}\left[-\underline{w_s}c_s - K_s\frac{\partial c_s}{\partial z}\right] = 0. \tag{10}$$

This states that downward settling of sediment ($\underline{w_s}c_s$) balances upward diffusion by turbulence ($K_s\frac{\partial c_s}{\partial z}$) for steady, uniform conditions. Note that, usually, $\partial c_s/\partial z < 0$. By taking the integral of equation **??**,

$$\begin{aligned}
-\underline{w_s}c_s - K_s\frac{\partial c_s}{\partial z} &= Constant; \\
-\underline{w_s}c_s - K_s\frac{\partial c_s}{\partial z} &= 0; \\
-\underline{w_s}c_s &= K_s\frac{\partial c_s}{\partial z}; \qquad (11)
\end{aligned}$$

where the *Constant* must be zero to have steady conditions at the bed.

Equation **??** can be integrated also

$$\begin{aligned}
\int_{z_a}^{z}\frac{\partial\overline{c_s}/\partial z}{\overline{c_s}}dz &= \int_{z_a}^{z}\frac{-\underline{w_s}}{K_s}dz; \\
\ln\left[\frac{\overline{c_s}}{c_s}\right] &= -\underline{w_s}\int_{z_a}^{z}\frac{1}{K_s(z)}dz. \qquad (12)
\end{aligned}$$

Recall that near the bed, the eddy viscosity for momentum increases linearly with z; $K_m = \kappa u_* z$. Assume that the eddy diffusivity for mass also follows this form near the bed;

$$K_s = \alpha K_m = \alpha\kappa u_* z \tag{13}$$

where α is a constant of proportionality to relate the eddy diffusivity to eddy viscosity. At low concentrations $\alpha \approx 1$. For high concentration suspensions, the value of $\alpha \approx 1.35$ is used.

Suspended Sediment Profiles

Near the bed, therefore, for steady uniform flow; we can use equations **?? ??** to solve for c_s as a function of z:

$$\frac{c_s}{c_a} = \left[\frac{z}{z_a}\right]^{-P/\alpha}; \tag{14}$$

where P is the Rouse parameter ($P = w_s/\kappa u_*$). The reference height, z_a is some height where concentration (c_a) can be specified. Equation **??** shows that sediment concentration drops away from the bed, and that the gradient will be steeper for large values of P. Low values of P (fine-grained sediment, intense turbulence) imply a more well-mixed sediment profile.

To get profiles that are valid throughout a thicker region of the bottom boundary layer, different values of the eddy diffusivity must be used. One simplifying assumption that is often made is that the eddy diffusivity profile is parabolic; $K_s = \alpha\kappa u_* z\,(1 - z/h)$, where h is water depth (shallower flows), or (for deeper flows) thickness of the bottom boundary layer. Using a parabolic eddy viscosity in equation **??** yields the *Rouse Profile*:

$$\frac{c_s}{c_a} = \left[\frac{z\,(h - z_a)}{z_a\,(h - z)}\right]^{-P/\alpha}. \qquad (15)$$

Close to the bed, equations **??** and **??** give similar values, but as $z \rightarrow h$, $c_s \rightarrow 0$ for the Rouse profile (equation **??**).

At high concentrations $(1 - c_s < 1)$, conservation of mass (equation **??** should be rewritten to take both water mass and sediment mass into account. This yields a correction to equation **??** of the form

$$\frac{c_s}{1 - c_s} = \frac{c_a}{1 - c_a}\left[\frac{z\,(h - z_a)}{z_a\,(h - z)}\right]^{-P/\alpha}. \qquad (16)$$

Choosing Reference Height

To use any of these suspended sediment profiles, you have to be able to specify a value of sediment concentration for some height. The reference height, z_a can be a location at which you have some idea of concentration from data or from theory. Yalin proposed that, near the bed at steady state, concentration should increase with excess shear stress, $c_s \sim S = \frac{\tau_b - \tau_{cr}}{\tau_b}$. One problem with this is that concentration would increase indefinately as excess shear stress increased. **?** proposed

$$c_a = \frac{c_b \gamma_0 S}{1 + \gamma_0 S}, \qquad (17)$$

where γ_0 is a constant. They use a reference height, z_a equal to the saltation height above the bed.

Mixed-grain Size Beds

For mixed grain sized beds and suspensions, the range of sediment types are separated into a finite number of size classes or sediment types, represented by the subscript j: $c_s = \sum c_{sj}$. At low concentrations, the sediment types do not interact with each other, and each class can be treated independently. The profile for each depends on that sediment class' Rouse number, $P_j = \underline{w_{sj}}/\overline{\kappa u_*}$. The reference concentration for each class depends on the fraction of that sediment type on the bed (f_j); $c_{aj} = f_j c_b \gamma_0 S/\,(1 + \gamma_0 S)$ (**?**).

Application to Coastal Environments

In shelf, nearshore, and estuarine flows, the shape of the eddy diffusivity should often account for wave-current interaction, and the influence of stratification from gradients in suspended sediment concentrations on turbulence. This gives more complicated forms of eddy diffusivity, so that equation **??** must be solved using numerical methods.