MS698–3: Sediment transport processes in coastal environments
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Lecture 7: Form drag and skin friction

- Form drag, skin friction, and total shear.
- Flow field around bedforms.

Class business

- Problem set due Thursday, 2PM
- This Thursday: Curran, K.J., P.S. Hill, and T.G. Milligan. 2002. Fine-grained suspended sediment dynamics in the Eel River flood plume. Continental Shelf Research, 22(17):2537–2550.

Materials used:

- Smith and McLean (1977); Li and Amos (2001)
- van Rijn, L.C. **Principles of sediment transport in rivers, estuaries, and coastal seas**; Chapter 6.

Handouts

- Schematic of flow field around bedform.

Bedforms influence flow structure and shear stress. The increased roughness created by the bedform acts to increase total bed shear stress in the region away from the bed (the *outer flow*). Form drag acting on the bedform, however, decreases the ability of flow shear to impact sediment grains on the bed, by decreasing the *skin friction* component of the total shear stress.

Bedforms

First, lets summarize the types of bedforms found in coastal environments.

- *Ripples* are the smallest type of bedform. They tend to be steep ($\lambda/\eta \sim 8-12$). The wavelength of ripples scales with grain diameter ($\lambda \approx 535D$), and for wave-dominated ripples, with the orbital diameter of the wave ($\lambda \sim d_o$).
- *Dunes* are larger bedforms, and their heights are limited by water depth ($\eta \leq h/4$); and are less steep than ripples ($\lambda/\eta \sim 14-30$).
- *Antidunes* occur infrequently in coastal flows, usually where water depth is small and flows are supercritical ($Fr > 1$, where Fr is Froude number).

The schematic on your handout identifies different parts of bedforms including the stoss and lee sides, the crest, the trough. Bedforms in unidirectional flows tend to be *asymmetric*, with a clearly defined stoss and lee slope. Under strong oscillatory flows (waves), bedforms tend to be *symmetric*.

Flow field around bedforms

The presence of a bedform alters the flow field surrounding it. In general, depth-averaged flow will be faster above the crest of the bedform, and slower above the trough. Depending on the shape of the ripple, and the intensity of the flow field, the flow may separate at the crest (most likely when the crest is sharp, and the flow is fast). The schematic on your handout identifies parts of the flow field surrounding a ripple or dune.

- The *outer flow* or *interior flow* is the part of the flow field that is far enough away from the bedform that it does not feel spatial variations created by the bedform. The roughness felt in the outer flow is dominated by the bedform (as discussed in an earlier lecture: $k_s \approx 27.7\eta^2/\lambda$; $z_0 \approx \eta^2/\lambda$, Grant and Madsen (1982)).

- The *internal boundary layer* is the portion of the flow adjacent to the bedform, where the flow field and shear stress vary spatially. For flows that separate at the ripple crest, the internal boundary layer grows downstream of the *reattachment point*. The roughness felt in the internal boundary layer is dominated by the sediment grains or sediment-transport components of roughness, $k_s \sim D$ or $k_s \sim \delta_B$, because the internal boundary layer does not really feel the presence of the bedform.

- The *wake* separates the internal boundary layer from the outer flow, and diffuses outward from the *separation point* - where the flow separates.

- In flows that separate, a *recirculation zone* exists between the separation and reattachment points. Flow is reversed in this section, and transport may actually be directed upstream.

- The *reattachment point* separates the recirculation zone from the internal boundary layer. Because flows separate here, it makes sense that skin friction shear stress felt at the reattachment point is zero.

Skin friction

From the last point, it follows that the shear stress felt at the bed (called the *skin friction shear stress*, τ_{sf}) is not equal to the spatially-averaged total shear stress (the *bed shear stress*, τ_b). The difference between the skin friction shear and the total shear is the *form drag* τ_{fd}.

$$\tau_b = \tau_{sf} + \tau_{fd} \tag{1}$$

The skin friction component is less than the total shear stress ($\tau_{sf} \leq \tau_b$). Only for a flat bed will the skin friction shear stress equal the total shear stress. The strength of the skin friction and form drag components of shear stress vary over the wavelength of a bedform. The skin friction shear stress acts tangential to the local bed surface.

Form drag on bedforms

As the flow field crosses a bedform, it goes from a region of high pressure (over the trough) to low pressure (over the crest). These pressure differences set up a horizontal drag force felt by the flow:

$$F_D = \frac{1}{2}\rho C_D u^2 A_D; \tag{2}$$

where F_D is the drag force, ρ is fluid density, C_D is a drag coefficient that depends on bedform shape, u is a velocity scale, and A_D is the cross-sectional area of the bedform that protrudes into the flow ($A_D = \eta \times width$). The drag force can be expressed as a shear stress (force / area; area = $\lambda \times width$):

$$\begin{aligned} \tau_{fd} = \frac{F_D}{A_F} &= \frac{1}{2}\rho C_D u^2 \frac{A_D}{A_F}; \\ &= \frac{1}{2}\rho C_D u^2 \frac{\eta}{\lambda}. \end{aligned} \tag{3}$$

The velocity scale in equations 2 and 3 should represent the average velocity of the flow that would be present if the bedform were not there. Depth-averaging over the height of the ripple gives

$$\begin{aligned} u = \langle u \rangle_{BF} &= \frac{1}{(\eta - z_0)} \int_{z_0}^{\eta} \frac{u_*}{\kappa} \ln \frac{z}{z_0} dz; \\ &= \frac{u_*}{\kappa}\left(\ln \frac{\eta}{z_0} - 1\right); \qquad (4) \\ &= \frac{u_{*sf}}{\kappa}\left(\ln \frac{\eta}{z_{0,sf}} - 1\right), \qquad (5) \end{aligned}$$

where κ is von Karman's constant (0.408). The shear stress and roughness acting within the internal boundary layer are used to get from equation 4 to equation 5, because we are depth-averaging flow velocity near the bed. The shear stress felt in that region is the skin-friction shear, define the skin-friction shear velocity to be $u_{*sf} = \sqrt{\tau_{sf}/\rho}$. The roughness felt in this region would be the "inner roughness" and would not be influenced by bedform - scale features. Define $z_{0,sf}$ to be the maximum of the sediment roughness ($z_{0,gr}$) and the sediment-transport roughness ($z_{0,st}$).

The drag coefficient in equations 2 and 3 is determined by the shape of the bedform and tends to be about $C_D = 0.2$ for asymmetric bedforms, and $C_D = 1$ for symmetric bedforms.

Combining equations 3 and 5, and using the definition of skin friction shear velocity:

$$\begin{aligned}\tau_{fd} &= \frac{1}{2}\rho C_D \left(\frac{u_{*sf}}{\kappa}\right)^2 \left[\ln \frac{\eta}{z_{0,sf}} - 1\right]^2 \frac{\eta}{\lambda}; \\ &= \frac{1}{2} C_D \frac{1}{\kappa^2} \left[\ln \frac{\eta}{z_{0,sf}} - 1\right]^2 \frac{\eta}{\lambda} \tau_{sf}. \qquad (6)\end{aligned}$$

Partitioning of shear stress

We can rearrange equations 1 and 6 to express the ratio of total shear to skin friction shear;

$$\begin{aligned}\tau_b = \tau_{fd} + \tau_{sf} &= \tau_{sf}\left(1 + \frac{1}{2} C_D \frac{1}{\kappa^2} \left[\ln \frac{\eta}{z_{0,sf}} - 1\right]^2 \frac{\eta}{\lambda}\right); \\ \frac{\tau_b}{\tau_{sf}} &= 1 + \frac{1}{2} C_D \frac{1}{\kappa^2} \left[\ln \frac{\eta}{z_{0,sf}} - 1\right]^2 \frac{\eta}{\lambda}. \qquad (7)\end{aligned}$$

The skin friction shear stress in equation 7 represents a spatial average of skin friction, averaged over a bedform wavelength. The ratio, τ_b/τ_{sf}, equals $\approx 2 - 3$ for many natural systems, so that the skin friction shear stress may account for less than a half of total shear stress. This implies that the presence of bedforms drastically alters the magnitude of the shear stress that acts on sediment grains.

Using τ_{sf} for transport equations

The skin friction shear stress defined by equation 7 represents the spatial average over the wavelength of a bedform. Remember that skin friction shear stress is zero at the reattachment point. It increases toward the crest of the bedform, where it reaches its maximum ($\tau_{sf,max}$), see Wiberg and Nelson (1992) for more discussion. Nielsen (1992) estimates the value of skin friction shear velocity at the crest of a ripple to be the *enhanced shear velocity*:

$$u_{*sf,max} = u_{*sf} \Big/ \left(1 - \pi\frac{\eta}{\lambda}\right). \qquad (8)$$

In the vicinity of the reattachment point, therefore, skin friction shear stress will not exceed the critical shear stress for motion. It therefore follows that sediment transport will occur on only part of the bedform, and will be strongest near the bedform crest, where $\tau_{sf} \approx \tau_{sf,max}$. There could easily be a case where the spatially-averaged skin friction shear does not exceed critical shear stress, while the maximum skin friction shear does exceed critical shear stress (possible for $\tau_{sf,avg} < \tau_{cr} < \tau_{sf,max}$).

Most bedload formulas, like those we covered last week, state that sediment flux (q_{BL}) is proportional to $\tau_{sf}^{3/2}$. Using the spatially averaged skin friction shear stress is likely to underestimate sediment flux. Applying the bedload formula using the maximum skin friction shear stress would likewise overestimate bedload.

Other Methods for Partitioning Drag

Smith and McLean (1977) present this material; and take the development a step further to consider flow fields around “nested” bedforms; such as would be the case if ripples stood atop dunes. I prefer treatments following Smith and McLean (1977) because they is based on a force balance and physically-based parameters (bedform height, bedform wavelength).

Many researchers (for example Grant and Madsen, 1982) instead use form drag partitions based on separating a skin-friction roughness (${k_s}^{'}$) component from the total roughness.

$$k_s = {k_s}^{'} + {k_s}^{''}; \tag{9}$$

where k_s is total roughness, ${k_s}^{'}$ is skin-friction roughness, and ${k_s}^{''}$ is form-drag roughness. Van Rijn presents his methodology for this in his Chapter 6, and Li and Amos (2001) provide a thorough discussion of their methodology for unidirectional, oscillatory, and combined flows. Li and Amos (2001) follow Grant and Madsen (1982) and define a hierarchy of roughness types

$$\begin{aligned} k_{bg} &= 2.5D; \\ k_{bt} &= 180\,(2.9D)\,(\tau_{*cws} - \tau_{*cr})^{(3/4)}; \\ k_{br} &= 27.7\eta^2/\lambda; \end{aligned} \tag{10}$$

where k_{bg} is the grain roughness (they call it skin-friction roughness), k_{bt} is sediment transport roughness (they call it bedload roughness), and k_{br} is bedform roughness. Each roughness type is used to estimate a shear stress through a related hydraulic roughness ($z_0 = k_b/30$), and combined wave-current friction factor ($f_{cw} = f\,(z_0, u_{*w}, u_{*c})$) using $\tau_b = 0.5\rho f_{cw} {u_{100}}^2$; where u_{100} is the velocity 1 meter above the bed.

References

Grant, W. D. and Madsen, O. S. (1982). Movable bed roughness in unsteady oscillatory flow. *Journal of Geophysical Research*, 87(C1):469–481.

Li, M. Z. and Amos, C. L. (2001). SEDTRANS96: the upgraded and better calibrated sediment-transport model for continental shelves. *Computers and Geosciences*, 27(6):619–645.

Nielsen, P. (1992). *Coastal bottom boundary layers and sediment transport.* World Scientific, Singapore.

Smith, J. D. and McLean, S. R. (1977). Spatially averaged flow over a wavy surface. *Journal of Geophysical Research*, 82(12):1735–1746.

Wiberg, P. L. and Nelson, J. M. (1992). Unidirectional flow over asymmetric and symmetric ripples. *Journal of Geophysical Research*, 97:12745–12761.