MS698–3: Sediment transport processes in coastal environments
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Lecture 6: Bedload Transport

- Bedload vs. suspended load.
- Form drag, skin friction, and total shear.
- Bedload equations: Meyer-Peter and Muller; Yalin; Einstein; Bagnold.

Class business

- Problem set hints on-line.
- This Thursday: problem set review; Office hours 11:30 - 1:00 Monday.

Materials used:

- Nielsen, P. **Coastal Bottom Boundary Layers and Sediment Transport**, Chapter 2.3, Steady bed-load and sheet flow.
- Raudkivi, A.J. **Loose Boundary Hydraulics**, Chapter 7: Sediment transport.
- van Rijn, L.C. **Principles of sediment transport in rivers, estuaries, and coastal seas**; Chapter 7.2. Bed-load transport.

Handouts

- Yalin and MPM equations fit to Φ vs τ_{*sf}.
- Einstein equation.
- Einstein probability function.

Thresholds for motion, bedload, suspended load

In the last lecture we considered the threshold of motion and found that $\tau_{*cr} = \tau_{cr}/[(\rho_s - \rho)\, gD] = f\,(R_* = u_* k_s/\nu)$, where τ_{*cr} is a non-dimensionalized critical-shear stress for motion, and R_* is the roughness Reynolds number. When the bed shear stress acting on the particles (τ_b) exceeds τ_{cr}, sediment can be mobilized by the fluid. At low positive values of *excess shear stress* (see equation 2 below), sediment will roll along the bed. As excess shear stress increases, sediment will begin to hop or saltate a few grain diameters above the bed. As flow intensity continues to increase, turbulent fluctuations in the velocity field will intensify, and eventually they become energetic enough to carry sediment far from the bed into suspension.

The differentiation between bedload and suspended load lies in the frequency of contact with the sediment bed, with bedload being in frequent contact with other sediment grains on the bed. This definition follows that of Bagnold; and includes saltating sediment in the bedload fraction.

If sediment is too coarse to be readily suspended in the flow; it may be transported as *grain flow* or *sheet flow* under very energetic shear stresses.

Thresholds for transport, bedload, and suspended load can be phrased in term of either *transport stage*, T_*, or *excess shear stress*, S;

$$T_* = \frac{\tau_b}{\tau_{cr}}; \quad (1)$$

$$S = \frac{\tau_b - \tau_{cr}}{\tau_{cr}}. \quad (2)$$

The following transport thresholds have been established

No Motion	$T_* < 1$	
Rolling	$1 < T_* < 1.5$–2	
Saltation	$2 < T_*$	
Suspension	$1 < T_*$	and $w_s < 2.5u_*$

Bedload Layer

The desire to quantify bedload stems from two goals. First, in some areas bedload dominates total sediment transport. In marine environments - this is usually confined to sandy nearshore or shoal areas, with suspension dominating in (all) muddy environments and in finer-sand environments (estuarine channels and shelves) Secondly, saltating sediment can serve as a roughness element felt by the flow, as discussed in lecture 4.

Bedload layers can be characterized by their thickness ($\delta_B \approx 2-5$D, L), concentration of sediment (c_s, either mass concentration M/L^3; or volumetric concentration L^3/L^3), and sediment flux (q_{BL}, either mass flux $M/(LT)$ or volumetric flux $L^3/(LT)$). Total sediment flux (suspended and bedload) at a point is $q_s = \int_0^h c_s u_s dz$, where c_s is the sediment concentration, and u_s is the horizontal sediment velocity. For suspended flows, $u_s \approx u$, the sediment velocity is about the same as the fluid velocity, but bedload particles are slower than surrounding fluid. The total bedload flux is $q_{BL} = \int_0^{\delta_B} c_s u_s dz$, or

$$q_{BL} = \langle c_s \rangle_{BL} \langle u_s \rangle_{BL} \delta_B; \quad (3)$$

where $\langle c_s \rangle_{BL}$ is a representative concentration, and $\langle u_s \rangle_{BL}$ is representative velocity of the bedload layer.

The bedload transport rate is often non-dimensionalized from the volumetric sediment flux:

$$\Phi = q_{BL} \left[\frac{\rho_s - \rho}{\rho} \, gD^3 \right]^{-1/2}. \quad (4)$$

A force balance argument (see Wiberg and Rubin, 1989) shows that the height of the bedload layer depends on shear stress and grain size, and is usually about 2–5 × D

$$\frac{\delta_B}{D} = \frac{a_1 T_*}{1 + a_2 T_*}; \qquad (5)$$
$$where\ a_1 \approx 0.6; \qquad a_2 \approx 0.2.$$

where a_1 is constant, and $a_2 \approx 0.2$ depends on sediment size.

Estimates of Bedload Flux

Researchers have been developing methods for estimating bedload flux for over 100 years. We'll discuss four methods for estimating bedload; the empirical Meyer-Peter and Muller equation, the more physically and stochastically-based Einstein equation, the process-based (but less widely used) Yalin equation, and the Bagnold equation for sheet flow.

Form drag and skin friction: All of these have sought to relate sediment flux (q_{BL}) to bed shear stress and sediment size, and have relied on data from flumes. One difficulty with developing bedload relationships has been that the total bed shear stress (τ_b), easily estimated for a flume, includes two components; the *form drag* (τ_{fd}) that acts on bedforms, and the *skin friction* (τ_{sf}) that acts on the sediment grains themselves. We'll discuss this more next week, but in order to estimate the amount of total shear that acts on the sediment grains themselves, you have to be able to remove the form drag component. This requires knowledge of bedform height and wavelength. When the bedform correction is made, bedload transport (Φ) depends on non-dimensionalized skin-friction shear stress ($\tau_{*sf} = \tau_{sf}/[(\rho_s - \rho)\, gD]$).

Meyer-Peter and Muller Equation: The simplest bedload equation to use was developed by Meyer-Peter and associates, with their work starting in 1934 (see Meyer-Peter and Muller, 1948). They used flume measurements of a sediment bed made up of grains sized $D = 0.03 - 2.9$ cm, with varying sediment densities ($\rho_s = 1.3 - 4.2\ g/cm^3$), and considered both well-sorted and naturally-sorted material. They noticed that at high shear stresses, $q_{BL} \sim {\tau_b}^{3/2}$, and that bedload ceased at low shears ($\tau_* < 0.047$). They came up with

$$\begin{aligned} \Phi &= 0;\ \tau_{*sf} < 0.047; \qquad (6) \\ \Phi &= 8\left(\tau_{*sf} - 0.047\right)^{3/2};\ \tau_{*sf} > 0.047. \end{aligned}$$

Equation 6 represents a general bedload equation, where $\tau_{*cr} = 0.047$ is an "average" value of non-dimensionalized critical shear stress. Later researchers found that the constant (8 in equation 6) increases with increasing shear stress. An alternative to equation 6 is to include the actual critical shear stress of the grain size, and use a variable constant;

$$\begin{aligned}\Phi &= 0;\ \tau_{*sf} < \tau_{*cr}; \\ \Phi &= C_0\left(\tau_{*sf} - \tau_{*cr}\right)^{3/2};\ \tau_{*sf} > \tau_{*cr};\end{aligned} \tag{7}$$

where $C_0 \approx 6$; for $(1 < T_* < 3)$; $C_0 \approx 8$; for $(3 < T_* < 20)$; and $C_0 \approx 12$; for $(20 < T_*)$. Use the τ_{*cr} for the D_{50} of the flow in equation 7.

Einstein-Brown equation: A different approach was pursued by Einstein and others, who tried to estimate sediment transport flux by accounting for the probability that any sediment particle within a population would be mobilized by the (fluctuating) flow field (see Einstein, 1950). This therefore takes into account such things as the ability of small sediment grains to hide within pore-space, and the intensity of turbulent fluctuations at the bed. The relationship is posed in terms of a functionality between $\Psi = \tau_*^{-1}$ and Φ. The functional form of the Einstein-Brown equation is summarized on the handout, and it is not unusual to implement the transport formula by using this graph. Van Rijn provides a version of the Einstein equation

$$\Phi = \frac{1}{A_*}\left(\frac{P}{1-P}\right);\ A_* = 43.5 \tag{8}$$

where P is the probability of a grains being lifted, as estimated by a relationship based on $P = f\left(\tau_*\right)$ (see handout).

Yalin Equation: This is the most mechanistic of the equations that we will consider. Yalin tried to estimate the three parts of the definition of sediment flux (equation 3) by finding scaling relationships for $\langle c_s\rangle_{BL}$, $\langle u_s\rangle_{BL}$, and δ_B. After these were scaled by physical parameters, he collected all of the constants into a few coefficients that could be fit using empirical data. We've already discussed a scaling for δ_B in equation 5. He argued that sediment concentration should scale with excess shear stress ($\langle c_s\rangle_{BL} \sim S$). Sediment velocity should be less than fluid velocity, but depend on the velocity structure within the saltation layer; which depends on depth averaged velocity $u(z) \approx \frac{u_*}{\kappa}\ \ln\left(z/z_0\right)$. Carrying this argument forward, Yalin arrives at

$$\begin{aligned}q_{BL} &= a_1 D S u_*\left[1 - \frac{1}{a_2 S}\ln\left(1 + a_2 S\right)\right]; \\ a_1 &= 0.635; \\ a_2 &= 2.45\left(\rho/\rho_s\right)^{0.4}\ \sqrt{\tau_{*cr}},\end{aligned} \tag{9}$$

where $a_1 = 0.635$ is a constant for all sediment and all fluids, a_2 is a constant that depends on sediment type and fluid type. This can be rewritten in non-dimensional form

$$\Phi = a_1 S\sqrt{\tau *}\left[1 - \frac{1}{a_2 S}\ln\left(1 + a_2 S\right)\right]. \tag{10}$$

Bagnold Equation: Bagnold (1966) approached the bedload problem by estimating the forces needed to move an entire *layer* of the bed relative to underlying layers, instead of the particle force balance that we considered last week. This type of transport is termed *grain flow* or *sheet flow.* The force required for this would be

$$\begin{aligned} F &= \tan\Psi_o\left(\rho_s-\rho\right)gV_s; \\ &= \tan\Psi_o\left(\rho_s-\rho\right)gc_sA\delta_B \end{aligned} \tag{11}$$

where $\tan\Psi_o$ is the *coefficient of internal friction.* Normalizing by area (A), $F/A=\tau_b$, and incorporating a velocity term on each side of equation 11 gives

$$\tau_b u_s = q_{BL} g\left(\rho_s-\rho\right)\tan\Psi_o. \tag{12}$$

Bagnold uses a scaling for velocity based on assuming planar beds, and hydraulically rough flow ($z_0/D=30$), so that the velocity at the height of the grain diameter is $u(z=D)\approx 8.5u_*$. Further defining the *efficiency factor,* e_b to represent the ratio of the flows capacity to do work to the amount of work done to move sediment, and using $\tau_b=\rho {u_*}^2$; Bagnold solves for q_{BL}:

$$q_{BL}=\frac{8.5e_b\rho {u_*}^3}{\left(\rho_s-\rho\right)g\tan\Psi_o}, \tag{13}$$

where $e_b/\tan\Psi_o$ must be empirically set using bedload measurements. The efficiency factor, e_b, will depend on the fluid (air or water) in which transport occurs, and ranges from $e_b\approx 0.1-0.2$ in water. The dynamic coefficient of friction, $\tan\Psi_o$ depends on sediment characteristics, and is about $\tan\Psi_o\approx 0.6$ for naturally shaped sediment. This relationship is often used in nearshore systems where sheet flow transport often seems to dominate. Note that there is no τ_{cr} in equation 13, so that bedload transport is predicted even when bed shear stress does not exceed critical shear stress. This makes equation 13 unsuitable for non-energetic environments. The Bagnold relationship is sometimes called an *energetics approach*; because he tries to parameterize sediment transport as a function of flow energy.

Comparing Bedload Formulas

both the Meyer-Peter and Muller, and Yalin formula go to relationships of the form $\Phi\sim\tau_{*sf}^{3/2}$ as $\tau_{*sf}\to\infty$, whereas the Einstein formula approaches $\Phi\sim 8\tau_{*sf}$. Meyer-Peter and Muller approaches $\Phi\approx 8\tau_{*sf}^{3/2}$ (using the classic equation 6), while Yalin approaches $\Phi\approx 12\tau_{*sf}^{3/2}$. Bagnold's equation 13 also follows $q_{BL}\sim{\tau_b}^{3/2}$, though it should not be used for low- to marginally energetic environments.

For real-world applications, uncertainties about the shear stress to use (τ_b vs. τ_{*sf}) usually outweigh the uncertainties in the actual bedload formula. In the next lecture we'll cover Smith and McLean's method for separating form drag and skin friction from total shear stress.

References

Bagnold, R. (1966). An approach to the sediment transport problem from general physics. Technical report, Geological Survey Professional Paper 422-I, Washington, D.C.

Einstein, H. (1950). The bed-load function for sediment transportation in open channel flow. Technical report, Technical Bulletin No. 1026, U.S. Department of Agriculture, Washington, D.C.

Meyer-Peter, E. and Muller, R. (1948). Formulas for bed-load transport. In *Proceedings of the 2nd conference*, page 39.

Wiberg, P. L. and Rubin, D. M. (1989). Bed roughness produced by saltating sediment. *Journal of Geophysical Research*, 94:5011–5016.