MS698–3: Sediment transport processes in coastal environments
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Lecture 2: Settling Velocities

- Force balance on a settling particle.
- Stoke's law.
- Dietrich's settling curves.
- Complicating factors.

Source materials

- Rijn, Leo C. van; **Principles of sediment transport in rivers, estuaries, and coastal seas**, Section 3.2.5: Particle fall velocities. Aqua Publications, 1993.
- Dietrich, W.E. 1982. Settling velocities of natural particles. *Water Resources Research*, 18(6): 1615–1626.

Handouts

- Properties of water and air.
- Drag coefficient vs. particle Reynold's number (C_D vs R_D).
- Dietrich's figure 8: W_* vs. D_* as function of CSF.

Class business

- *Error in Jan. 16 notes:* Page 5. The standard deviation formula should be $\sigma = -\left(\phi_{84} - \phi_{16}\right)/2$.
- *Error in Jan. 16 handout:* There is an error in the cumulative distribution plot for the sediment trap material; it is plotted one phi-size off.
- The reading for Thursday (Jan. 23) can be downloaded from Elsevier: Drake, D.E., R. Eagenhouse, W. McArthur. 2002. Physical and chemical effects of grain aggregates on the Palos Verdes margin, southern California. *Continental Shelf Research* 22: 967–986.

Definition: The *settling velocity* or *fall velocity* or *terminal velocity* (w_s) of a sediment particle is defined as the rate at which the sediment settles in still fluid. It is diagnostic of grain size, but is also sensitive to the shape (roundness and sphericity) and density of the grain as well as to the viscosity and density of the fluid. It integrates all of these into a key transport parameter.

Viscosity: The *dynamic viscosity* (μ, *mass/length/time*) and *kinematic viscosity* ($\nu = \mu/\rho$, $length^2/time$) of the fluid influence settling velocities. They depend on temperature. Values for viscosity of air, pure water, and seawater are given on the handout.

Force balance on settling particles.

Advances in predicting sediment settling velocities have come from a school of theoretical and laboratory work. Weight, buoyancy, and drag act on particles in a fluid.

Gravity: The *net gravitational force* is the difference between weight and buoyancy ($F_G = \rho_s V g$; $F_b = \rho V g$) where ρ_s and ρ are the sediment and fluid densities, V is the volume of the sediment particle, and g is the acceleration of gravity (g $\approx 980 cm/s^2 = 9.8 m/s^2$). If we define positive as downward, the net gravitational force on the particle is therefore

$$F_g = F_G - F_b$$

$$F_g = (\rho_s - \rho) V g \quad (1)$$

$$\text{For a sphere:} \quad F_g = \frac{\pi}{6} D^3 (\rho_s - \rho) g. \quad (2)$$

Note: Force (mass × acceleration) has units of Newton's (S.I.) or dynes; where 1 N = 1 kgm/s^2, 1 dy = 1 gcm/s^2, and 1 N = $10^5 dy$.

For a sphere, the volume, $V = \frac{4}{3}\pi r^3 = \frac{\pi}{6} D^3$, where D is grain diameter. The net gravitational force on a sphere is therefore Equation 2. The net gravitational force is downward (positive) is the sediment is more dense than the fluid, and upward (less than zero) is the sediment is less dense than the fluid.

Imagine a sediment grain initially at rest. Once the gravitational force begins to accelerate the particle, its velocity increases. The sediment, now moving through the fluid, will feel a frictional drag force, F_D, that is proportional to the square of the relative velocity of the particle in the fluid. F_D will increase as the particle accelerates, until, eventually, the drag force exactly balances the net gravitational force. At that point, the sediment has reached its terminal fall velocity.

Drag: The *drag force* depends on the particle's shape, size, and relative velocity, and on the fluid's density and viscosity.

$$F_D = \frac{1}{2} \rho C_D u^2 A, \quad (3)$$

where u is the velocity of the particle relative to the fluid, and A is the cross-sectional area of the particle perpendicular to its trajectory. The drag coefficient, C_D, is a non-dimensional number that depends on the shape of the particle, the fluid's kinematic viscosity, and grain size. You can see from Equation 3, that the drag force will increase with the square of the velocity of a settling particle.

The net force on the particle will be the difference between the net gravitational force and the drag; $F_g - F_D$. Once the drag force has increased to the point where it exactly balances the gravitational force ($F_D = F_g$), the particle will cease to accelerate and will have attained its terminal velocity. At this point the velocity in Equation 3 equals the settling velocity, $u = w_s$. By equating the drag law (Equation 3) with the definition of net gravitational force (Equation 1), and substituting w_s for u we get a general law for settling:

$$w_s = \sqrt{\frac{\rho_s - \rho}{\rho} \frac{g}{C_D/2} \frac{V}{A}} \, . \tag{4}$$

Drag coefficient: To go further with this, we need to think about the drag coefficient, C_D. Drag coefficients have been defined for different shaped objects, and laboratory studies have been used to plot drag coefficients for a range of flow characteristics. For the case of settling of natural grains, one approach is to use theoretical and empirical relationships obtained for perfect spheres, and then adjust them to account for the range of natural sediment shapes.

Stoke's Law

Stoke's Law of settling comes from simplifying Equation 4 for the case of a small sphere. The drag coefficient of spheres has been found to be a function of a non-dimensional number, the *particle Reynolds number*, $R_D = \frac{uD}{\nu}$, where u, D, and ν are velocity, sphere diameter, and kinematic viscosity. The particle Reynolds number is used to indicate whether the boundary layer around a particle is turbulent or laminar, and the drag exerted will depend on this. The handout provides the relationship between drag coefficient and particle Reynolds number for spheres.

For small particle Reynolds numbers ($R_D < 0.5$, $R_D \ll 1$);

$$C_D \approx \frac{24}{R_D} = \frac{24\nu}{uD}. \tag{5}$$

By substituting the drag coefficient for a sphere (Equation 5) into Equation 4 we get a settling law for low-Reynolds number particles ($R_D \ll 1$);

$$w_s = \frac{(\rho_s - \rho)\, g}{12\mu} \frac{V}{A} D. \tag{6}$$

If we next assume that the particle is an ideal sphere, the geometrical terms simplify: $\frac{VD}{A} = \frac{2D^2}{3}$. This can be simplified to obtain *Stoke's Law of Settling* valid for small grains whose shape approximates a sphere:

$$\text{For spheres, } R_D < 0.5: \quad w_s = \frac{1}{18\mu} (\rho_s - \rho)\, g D^2. \tag{7}$$

For water ($\nu \approx 0.01 cm^2/s$), the particle Reynolds number is $R_D = \frac{uD}{\nu} \approx 100(s/cm^2)uD$. For this to be less than 0.5, $uD < 0.005 cm^2/s$. Even small grains going at small speeds will exceed this. For example, sediment sized 0.01cm = D has a settling velocity of about $w_s = 0.075$ cm/s. This would be out of the Stoke's Range ($Dw_s = 0.075cm^2/s > 0.005cm^2/s$). Sediment sized 3.5 ϕ is just barely in the Stoke's range; for such sediment, D=0.0088 cm and $w_s = 0.0053$cm/s; this gives $Dw_s \approx 0.0053cm^2/s$. So, in water, Stoke's range includes sediment sized 3.5 ϕ and finer. Stoke's Law is the basis for measuring "effective diameter" of settling particles. The settling velocity is measured, and then diameter of the equivalent sphere is backed out of Equation 7.

To summarize, inside Stoke's Range; ($R_D < 0.5$; $R_D << 1$); settling velocity increases with D^2 (Equation 7). Above Stoke's Range ($10^3 \leq R_D \leq 10^5$), there is a significant portion of the C_D vs. R_D curve in which C_D remains constant. Here, $w_s \; \alpha \; \sqrt{D}$.

Dietrich's settling curves.

Many natural particles are too coarse for Stoke's Law to hold, and natural particles are not usually spheres. Natural particles tend to have lower settling velocities than perfectly round spheres. Natural particles will tend to have lower settling velocities because both decreases in sphericity and increases in angularity tend to decrease settling velocities. More oblong particles (less spherical) tend to have lower settling velocities because (1) the larger-cross sectional area tends to be directed perpendicular to transport path; (2) flow separation (increases drag) is more likely to occur for non-spherical particles; and (3) oblong particles may rotate, follow wobbling paths, etc. as they settle. Angular particles also tend to have lower settling velocities than round ones; because the increased roughness of the particle surface increases drag for typically sized particles.

The traditional way to estimate settling velocity is to use Equation 4, assuming that the drag coefficient can be estimated by the relationship for spheres, and then apply a correction factor for deviations of roundness and angularity. To do this, you would guess a settling velocity, use that to get a Reynolds number ($R_D = \frac{w_s D}{\nu}$), use the Reynolds number and the graph on the handout to estimate a drag coefficient, and then calculate settling velocity using Equation 4. The revised settling velocity would be used in the next iteration, and the process could continue until settling velocity converges. Then, a correction factor would be applied to account for shape variations.

Dietrich (1982) noted that this is awkward, and suggested a more straightforward way of estimating settling velocities. He furthermore noted that many of us are particularly concerned with *natural particles* and restricted the data used in his analysis to settling velocities obtained for spheres and natural-like sediment particles. He proposed using other non-dimensional numbers (W_*, which includes settling velocity, and D_*, which included grain diameter) for mapping out the relationship between settling velocity and grain size. The non-dimensional settling velocity, W_* is the ratio of the particle Reynolds number to the drag coefficient; $W_* = \frac{4}{3}\frac{R_D}{C_D}$;

$$W_* = \frac{\rho {w_s}^3}{(\rho_s - \rho)\, g\nu}; \tag{8}$$

while the non-dimensional grain size ($D_* = \frac{3}{4} C_D {R_D}^2$) compares the drag and gravitational forces on the particle:

$$D_* = \frac{(\rho_s - \rho)\, g}{\rho \nu^2} D^3. \tag{9}$$

Dietrich replotted available data, and noted that natural sediments tend to vary more with respect to sphericity (as denoted by Corey Shape Factor) than they do with angularity. His Figure 8 (handout provided) gives a series of plots of W_* vs. D_* for naturally sized sediment, as a function of CSF.

Complicating Factors

The analysis above assumes that a single particle is settling in still water, and is not influenced by other particles in the water. It also assumes that the drag coefficient that parameterizes drag on the particle is reasonably well estimated by the drag coefficient for a sphere. In the marine environment, particularly where fine-grained silts and clays are present, these assumptions may not hold.

First, particles may flocculate and form large, less dense, groups of particles. The settling velocities of these will be larger than would be apparent if the grains remained disaggregated in the water column.

Secondly, at high concentrations, the return flow of water around a particle may create an upward drag on neighboring particles. In *hindered settling* regimes, these become large enough to keep sediment fluidized, and to prevent settling. Hindered settling is often accounted for by estimating an actual settling velocity, ${w_s}'$; where ${w_s}' = w_s\,(1 - c_s)^n$. Here, c_s is the volume-concentration of sediment, and n is a parameter that depends on particle Reynolds number (typically $n \approx 4.6 \cdot 2.3$); see van Rijn for citations.