MS554 Assignment 2: Numerical Calculations, Interpolation Methods

- Assigned Sept. 13, 2001
- Due Sept. 20, 2001
- Do 6 of the following 7 problems.

Floating Point Computations

1. Get the c-program *enquire.c*, and compile it and run it. If your machine as a c-compiler (usually gcc or cc) then, in unix, the command "sh enquire.c" should compile the program, run it, and put the output into a file named enquire.out. Print this file out. The code and more information is available at http://www.cwi.nl/~steven/enquire.html.

2. Recall that epsilon ϵ is the smallest floating point number that can be added to 1.0 such that $1.0 + \epsilon > 1.0$. Write a Fortran routine to estimate epsilon (ϵ), and apply it for both reals and double reals on your FORTRAN compiler. Compare it to the value given by enquire.c (if you did the first question). Rewrite the program as a matlab m-file, and see what you get for ϵ in matlab. Compare it to the value returned by >> *eps*. *Gerald and Wheatley; Chapter 0, Problems 21 and 29.*

3. Evaluate the following polynomial for x=1.07, using both chopping and rounding to three digits. Proceed through the polynomial from left to right, and then redo it, evaluating from right to left.

$$2.75x^3 \; - \; 2.95x^2 \; + \; 3.16x \; - \; 4.67 \qquad (1)$$

   Evaluating a polynomial in "nested form" is more efficient. Evaluate the equivalent polynomial using both rounded and chopped truncation:

$$((2.75x - 2.95) + 3.16)\, x - 4.67. \qquad (2)$$

   What are the absolute and relative errors of the answers that you got using the six methods (2 methods of truncation $\times$ 3 ways of evaluating the polynomial)? *Gerald and Wheatley; Chapter 0, Problems 24–26.*

Numerical Calculus

4. Estimate the rate at which water level changes at Gloucester Point, using the tide gage record that you obtained for Assignment 1. Do the derivatives as forward, backward, and centered. Produce a plot of the timeseries of $d\eta/dt$ using each of the three methods, where η is water level.

5. Numerically evaluate the integral

$$\int_{x=1.0}^{1.8} \cosh(x)dx. \tag{3}$$

Write matlab-m files to do the trapezoid rule and Simpson's rule, and apply them for successively smaller values of Δx until your answers converge to 4 significant digits. First, find $f(x) = \cosh(x)$ for each $x = 1, 1 + \Delta x, 1 + 2\Delta x, \cdots, 1.8$. Then apply the trapezoid rule and Simpson's rule to the results. Plot the answers that you get as a function of Δx, the graph should show that Simpson's rule converges more quickly. Plot the relative errors vs. Δx, using as the "exact answer" a value that you get from matlab's *trapz* function, i.e. ">> intcosh = trapz((1:0.00001:1.8), cosh(1:0.00001:1.8))". *Gerald and Wheatley; Chapter 5, Problems 40, 48.*

Interpolation Methods

6. Suppose that you are running a numerical model that needs as input water levels at Gloucester Point, VA, measured every 15 minutes. Write a linear interpolation routine to get water levels spaced 15 minutes apart from the hourly data that you downloaded for Assignment 1. Use matlab's spline function to estimate water levels 15 minutes apart. Produce a plot of time vs. hourly water level, and the two interpolated values. Produce a second plot or panel that shows the difference between the linear interpolations and the spline.

7. The following values are for $f(x) = 1/(x+2)$. Find values for f'(x), and f"(x) at x=1.5, 2.0, and 2.5 using a cubic spline function that approximates f(x). Compare to the true values to estimate the errors. Also compare to derivative values computed from central-difference formulas.

   f(1.0)= 0.333; f(1.5)= 0.286; f(2.0)= 0.250; f(2.5)= 0.222; f(3.0)= 0.200.

   Plot the values of f'(x) and f"(x) from the cubic splines and compare to plots of the true values.