Reformulation of Mine Scour Equations Using Observations from MBP Field Sites

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LONG-TERM GOALS

A major goal of the ONR Mine Burial Prediction (MBP) Program is to provide the operational Navy simple models for predicting subsequent mine burial by scour which work with a known and useful degree of accuracy in regions of strategic interest, defined initially as sandy inner shelves dominated by waves. In order to be useful under real world conditions, such models must be reasonably accurate and reliable but also simple and fast enough to execute in a practical, straightforward manner by the Fleet. Thus the favored approach must parameterize the complicated and computationally intensive details of localized mine scour. In response to the above needs of the operational Navy, the long-term goal of this project is to determine the practical utility of predicting scour related mine burial using a simple parameterized model based on readily available wave and sediment data.

OBJECTIVES

In this study, existing field data from the inner shelf collected near the Martha’s Vineyard Coastal Observatory (MVCO), Massachusetts, and offshore of Indian Rocks Beach (IRB), Florida, are being used to further improve parameterizations of the following components of parameterized burial models: (i) the characteristic time-scale of the scour process for cylindrical mines, (ii) the equilibrium depth of scour for such mines, (iii) relationships between depth of burial and percent area buried, and (iv) the functional dependence of these processes on the Shields parameter and the Keulegan-Carpenter number.

APPROACH

Our approach in predicting mine burial has been to critically examine established engineering relations for the depth of scour around seabed objects, such as:

\[ S = S_{eq} \left(1 - \exp\left(\frac{t}{T^*}\right)^{P}\right), \]

(e.g., textbooks by Whitehouse, 1998, and by Sumer and Fredsoe, 2002). In (1), \(S_{eq}\) is the equilibrium scour depth relative to the undisturbed far-field bed, \(T^*\) is the characteristic time-scale of the scour process, and \(P\) is a fitting coefficient of order 1 which depends mainly on the object’s geometry.

The text books referenced above both present the time-scale of scour as an empirical function, f(\(\theta\)), of the skin-friction Shields parameter just outside the object’s influence, normalized by dimensional...
parameters associated with the problem, namely object diameter (D), acceleration of gravity (g), specific gravity of sand (s), and median sand grain size (d):

$$T^* = f(\theta) \ D^2 \ (g(s-1)d^3)^{1/2}$$  \hspace{1cm} (2)

The Shield’s parameter is given by $\theta = (\tau_b/\rho)[g(s-1)d]^{-1}$, where $\rho$ is the density of water and $\tau_b$ is the magnitude of bed shear stress acting on sand grains away from the influence of the object. For pipelines, Sumer and Fredsoe (2002) found that (2) applies well to scour induced by either steady currents or waves without considering wave period, despite the fact that (2) does not depend on the frequency of oscillatory flow.

For the case of settling cylinders subject to scour, Whitehouse (1998) found $f(\theta) = 0.095 \ \theta^{-2.02}$, $P = 0.6$, and

$$S_{eq}/D = 0 \text{ for } (\theta/\theta_{cr})^{1/2} < 0.75 ,$$

$$S_{eq}/D = 1.15 \ (2 \ (\theta/\theta_{cr})^{1/2} - 1.5) \text{ for } 0.75 \leq (\theta/\theta_{cr})^{1/2} < 1.25 ,$$

$$S_{eq}/D = 1.15 \text{ for } (\theta/\theta_{cr})^{1/2} \geq 1.25 ,$$

where $\theta_{cr}$ is the critical Shields parameter for the initiation of motion of non-cohesive sand.

In Whitehouse (1998), (3) is applied to results for steady currents. For pipelines and pilings scoured by waves, Whitehouse (1998) and Sumer and Fredsoe (2002) have found $S_{eq}$ to be a function of both the Shields parameter, $\theta$, and the Keulegan-Carpenter number, $KC$, given by

$$KC = U T/D ,$$

where $U$ is the near-bed wave orbital velocity and $T$ is the wave period. The general trend found by both Whitehouse (1998) and Sumer and Fredsoe (2002) is that local scour due to waves (small KC) is somewhat smaller than the steady current value. A similar result regarding KC has been found for wave-induced scour around settling cylinders in laboratory experiments by Voropayev et al. (2003):

$$S_{eq}/D = 1.3 \ [1 - \exp(-0.06 \ (KC - 2))] [1 - \exp(-40(\theta - 0.018))] .$$

In further laboratory experiments on wave-induced scour around settling cylinders, Cataño-Lopera et al. (2007) also found a dependence on wave period with shorter period waves scouring less, although the result was not expressed as a simple function of KC:

$$B_{eq}/D = 0.8 \ \theta^{0.85} \text{ for } T < 4 \text{ sec, } B_{eq}/D = 1.6 \ \theta^{0.85} \text{ for } T > 4 \text{ sec} .$$

In (6), $B_{eq}$ is the equilibrium burial depth of the base of the cylinder relative to the far-field sediment water interface once the cylinder has settled to the base of its scour pit. By assuming free settling cylinders have a strong tendency to settle to the bottom of their scour pits, we consider $S_{eq}$ and $B_{eq}$ to be interchangeable.

**WORK COMPLETED**

Research during FY2007 has focused on consolidating and analyzing available field observations of
wave properties and scour induced mine burial from the MVCO and IRB MBP sites. The below results are presently being incorporated into a journal manuscript co-authored by Friedrichs, Trembanis and key providers of mine burial field data.

RESULTS

From analysis of MVCO and IRB field data, we have so far been able to identify 67 separate field measurements of percentage mine burial by depth (100*B_{eq}/D) at MVCO and IRB associated with 19 independent combinations of grain diameter (d), representative near-bed wave orbital velocity amplitude (U = 2^{1/2} \sigma_u) and wave period (T) for usable wave events. Wave events were considered usable for forcing further burial if they included the largest burst-averaged U measured up to that point during a given MBP field experiment. In the analysis presented here, it is assumed that wave events last long enough for B to reach B_{eq}.

The sources of near-bed orbital velocity wave measurements reported here are P. Howd (pers. comm.), P. Traykovski (pers. comm.), and Trembanis et al. (2007). The sources of the mine burial data collected at MVCO and IRB are Bower et al. (2007), Bradley et al. (2007), Mayer et al. (2007), Traykovski et al. (2007), Trembanis et al. (2007), Wolfson (2005), and Wolfson et al. (2007). The sources of grain size information are Richardson et al. (2004), Goff et al. (2005), and Bower et al. (2007).

Figure 1 displays observed percent burial by depth as a function of (i) the Shields parameter (with \( \theta \) calculated via Swart’s equation for wave friction factor as described by Trembanis et al., 2007) and (ii) the Keulegun-Carpenter number for all 67 cases of observed burial by depth. Although there is substantial scatter, there is a general tendency for percent burial to increase as a function of Shields Parameter over the range of \( \theta \approx 0.05 \) to 0.5 (Figure 1a). Furthermore, coarse and fine sand cases (stars and circles) seem to broadly follow the same general trend, as do observations from both MVCO and IRB (blue and red symbols). The relationship between burial and the Keulegun-Carpenter number is much less obvious and is particular inconsistent among fine and coarse cases (Figure 1b). This is in contrast to laboratory results that have concluded that KC is a key parameter in predicting depth of burial (Voropayev et al., 2003; Testik et al., 2007). It is possible that KC becomes less important relative to \( \theta \) as field scales are approached.

For \( \theta < \sim 0.5 \), the least-squares best-fit linear relationship, \( \pm 1 \) standard error (s.e.), between burial depth and slope for all points together is

\[
B_{eq}/D = 2.05 \pm 0.06 \theta, \tag{7}
\]

if one assumes \( B_{eq} = 0 \) when \( U = 0 \). A multiple linear regression involving both \( \theta \) and KC yields:

\[
B_{eq}/D = 2.24 \pm 0.16 \theta - 0.008 \pm 0.007 KC. \tag{8}
\]

Equation (8) further reinforces the conclusion that there is no significant dependence of scour-induced burial on the Keulegun-Carpenter number at the MBP field sites, especially since laboratory results suggest B/D should increase with KC, not decrease.
Figure 1. Observed burial by depth of inert cylindrical mines on the inner shelf as a function of (a) Shields parameter and (b) Keulegan-Carpenter number. All 67 observations are shown. Star = coarse sand, circle = fine sand, red = Indian Rocks Beach site, blue = Martha’s Vineyard Coastal Observatory. Observed burial by depth increases with Shields parameter for Shields parameter less than about 0.5. This is not a strong relationship between the Keulegan-Carpenter number and observed burial by depth.
Although there are 67 separate observations of burial depth available, there are actually just 19 independent combinations of grain diameter and wave properties for specific burial events. For some burial events there were multiple mines measured, and in some cases, the depth of an individual mine was measured using more than one method. This redundancy provides an opportunity to evaluate the likely accuracy of individual burial measurements, allows us to assess the likely variance in burial among separate mines during a given event, and makes it possible to fit weighted regressions to the entire pool of data that account for individual measurement uncertainty.

Among the 67 observations, there are 24 pairs of observations that document the burial of a given mine using two different techniques for evaluating burial. In each case, the pair of burial estimates are derived from the suite of (i) coverage of a mine’s optical sensors by sediment, (ii) the mine’s acoustic shadow determined from a nearby tower, (iii) change in the mine’s depth inferred from an internal pressure sensor, or (iv) change in its depth inferred from repeated multibeam surveys. The average absolute difference between the individual burial estimates and the mean of the pairs is 6.4±0.8%. Thus the very best case scenario is that the individual estimates are, on average, accurate to no more than about ± 5 to 7% percent burial by depth.

On 9 separate occasions, the burial depth of more than one mine was recorded in response to a wave event for a given grain size. The mean standard deviation among these replicate mine burial cases was 15.4±2.1%, suggesting that even if an individual burial observation were perfect, that one observation is likely to vary from the true mean burial for that given event by about 13 to 18%. The scatter for mine burial depth on coarse sand for a given wave event was larger than that for fine sand (22.7±3.0% vs. 11.7±1.0%).

After pooling all mine burial measurements with common forcings, 19 independent combinations of grain diameter and wave properties remain for specific burial events. It is useful to average all measurements for each of these unique events and plot each of these events as a function of Shields parameter (θ), with error bars on each data point representing ± 1 s.e., including the 6.4% error associated with the underlying individual measurements. The weighted best-fit linear relationship for unique events with θ < 0.5 then becomes (Figure 2)

\[
\frac{B_{eq}}{D} = 2.07 \pm 0.07 \theta . \tag{9}
\]

The corresponding best fits for various subsets of the data are

\[
\begin{align*}
\frac{B_{eq}}{D} &= 2.08 \pm 0.07 \theta \quad \text{(fine cases only)}, \\
\frac{B_{eq}}{D} &= 1.91 \pm 0.24 \theta \quad \text{(coarse cases only)}, \\
\frac{B_{eq}}{D} &= 2.01 \pm 0.11 \theta \quad \text{(MVCO cases only)}, \\
\frac{B_{eq}}{D} &= 2.13 \pm 0.04 \theta \quad \text{(IRB cases only)}.
\end{align*}
\]

(10a)  \quad (10b)  \quad (10c)  \quad (10d)

It is interesting to note that none of the best-fits for the various subsets are significantly different from the mean of all events given by (9). There are trends that burial depth as a function of the Shields parameter is slightly less for coarse sediment than fine and slightly less at MVCO than at IRB, but these differences are not statistically significant.
Figure 2. Observed burial by depth of inert cylindrical mines on the inner shelf as a function of Shields parameter. In this case observations have been pooled and averaged for 19 independent combinations of grain diameter and wave properties. Error bars represent ± 1 standard error around the mean of the underlying individual measurements. Star = coarse sand, circle = fine sand, red = Indian Rocks Beach site, blue = Martha’s Vineyard Coastal Observatory. Observed burial by depth increases with Shields parameter for Shields parameter less than about 0.5.

IMPACT/APPLICATIONS

Our earlier work with the Whitehouse scour equations has already impacted the strategy being taken by others to provide a working mine burial model for the operational Navy. Our MatLab formulation of the Whitehouse governing equations were passed on to Paul Elmore at NRL, who incorporated them into his prototype linked modeling system (Elmore et al., 2007), and to Alan Brandt and Sarah Rennie at Johns Hopkins, who have incorporated the equations into their probabilistic expert system (Rennie et al., 2007). Our formulation of the Whitehouse scour equations has also helped spur related applications of these equations to field and laboratory observations of wave-induced scour by others in the MBP program (Bower et al., 2007; Cataño-Lopera et al., 2007; Mayer et al., 2007; Testik et al., 2007; Wolfson et al., 2007). Development of our work has hinged on a close working association with field observationalists including Bower, Bradley, Howd, Mayer, Naar, Richardson, Traykovski, Wever, and Wolfson, amongst others.

TRANSITIONS

Our earlier formulations of the Whitehouse equations have been adopted by ONR-funded authors of prototype linked models and expert systems (see “Impact/Applications above) and are presently the
core of the scour-induced burial component of the MBP prediction package being transitioned to Naval Operations (Plant, 2005). Finally, our formulation has already influenced science outside of the MBP program, where the same equations have been applied by my colleagues to the study of scour-induced burial of marine artifacts (Trembanis et al., 2003; McNinch et al., 2006).

RELATED PROJECTS

The following recent projects involving Friedrichs also focus on coastal sediment transport:


REFERENCES


PUBLICATIONS


