Interaction of physical and biological processes in a microtidal estuary: an idealised model approach

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ABSTRACT

Introduction
In many estuaries and tidal channels characteristic (spatial and temporal) patterns of both suspended sediments and phytoplankton concentrations are observed. Observations of algae in the York river, a tributary to the Chesapeake Bay (VA.), show that during the winter-spring a strong algal bloom is often present in the mid reach of the mesohaline zone. During the summer a smaller bloom often occurs in the tidal fresh-mesohaline transition zone. From observations [Sin et al., 1999] it is concluded that phytoplankton growth is N-limited in the lower estuarine regions. Furthermore, sediment particles are often trapped at specific locations in the estuary, resulting in pronounced estuarine turbidity maxima (ETM), see Lin and Kuo [2001]. This results in strong light attenuation in these regions. Hence, the occurrence of the ETM can result in light-limited growth of benthic organisms.

In this contribution a simple model is presented and analyzed to gain more understanding about the coupled dynamics of the suspended sediments and phytoplankton concentrations. Approximate analytical solutions of the equations are constructed by making an expansion of the physical variables in a small parameter \( Z/H \), the ratio of the amplitude of the vertical tide and the undisturbed water depth. Next, some results are discussed and conclusions are given.

Model description
The geometry consists of an open channel with rectangular cross-section and a flat bed, whereas the width converges exponentially with a length scale \( L_b \) which is taken from observations. The water motion is modeled by the width-averaged shallow water equations. At the seaside the system is forced by a prescribed tidal elevation, whilst at the landside a river inflow is imposed. The width-averaged advection-diffusion equation is used to find the concentration profiles in the embayment. The density profile is prescribed diagnostically in both the horizontal and vertical direction. The following density profile is used:

\[
\rho = \rho_0(x) + \rho_1(x, z)
\]  

(1)
Here the first contribution on the right-hand side describes the observed gradual decrease of density from the sea to the river, and the second term reflects the observation that systems like the York river are always stably stratified. The stable stratification influences the eddy viscosity and diffusivity coefficient, here the formulation discussed in Van de Kreeke and Zimmerman [1990] is used.

Let $N$ denote the nutrient concentration and $P$ the phytoplankton population density. To model their dynamics, the following equations are used (see Huisman [2006] and references therein for an extensive discussion of the model equations):

$$\frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + w \frac{\partial N}{\partial z} = -\alpha \mu(N, I) P + \delta \alpha P + K_v \frac{\partial^2 N}{\partial z^2}$$  \hspace{1cm} (2a)

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + w \frac{\partial P}{\partial z} = \mu(N, I) P - m P - \nu \frac{\partial P}{\partial z} + K_v \frac{\partial^2 P}{\partial z^2}$$  \hspace{1cm} (2b)

where $\mu(N, I)$ is the specific growth rate of the phytoplankton as a function of nutrient availability $N$ and light intensity $I$, $m$ is the specific loss rate of the phytoplankton, $v$ is the phytoplankton sinking velocity, $K_v$ is the vertical turbulent diffusivity, $\alpha$ the nutrient content of the phytoplankton, $\delta$ is the proportion of nutrient in dead phytoplankton that is recycled, and $u$ ($w$) is the horizontal (vertical) velocity.

The specific growth rate $\mu(N, I)$ is modeled as

$$\mu(N, I) = \mu_{\text{max}} \min \left( \frac{N}{H_N + N}, \frac{I}{H_I + I} \right)$$

where $\mu_{\text{max}}$ is the maximum specific growth rate, $H_N$ and $H_I$ are the half-saturation constants for nutrient-limited and light-limited growth respectively. The light intensity $I$ decreases exponentially with depth according to Lambert-Beer's law.

As boundary conditions a constant nutrient flux is present at the riverine side and a no-flux condition for nutrients through the bottom and free surface is used. For the phytoplankton, we assume zero-flux boundary conditions everywhere.

To determine the specific growth rate $\mu(N, I)$, both $N$ and $I$ have to be known. To obtain $I$, the vertical distribution of sediment in the water column has to be known. In order to find these concentration...
profiles the coupled hydro- and concentration model is systematically investigated. The analysis is based on the fact that the parameter $\varepsilon = Z/H$ ($Z$ the tidal wave amplitude and $H$ the water depth) which measures the relative influence of nonlinear terms with respect to linear terms in the equation of motion, is usually a small parameter (for example in the York estuary $\varepsilon \sim 0.05$). Hence approximate solutions can be constructed by expanding the physical variables in power series of $\varepsilon$ and solving the equations at various orders of $\varepsilon$. By requiring that there is no net sediment flux in the estuary, these concentration profiles can be obtained. Using these profiles, the light intensity in the estuary can be calculated at any position.

![Salinity profile and vertical diffusivity](image1)

(a) Salinity profile and vertical diffusivity

![Nutrient profile](image2)

(b) Nutrient profile

![Sediment concentration profile](image3)

(c) Sediment concentration profile

![Specific growth rate](image4)

(d) specific growth rate $\mu$

Figure 2: Plots of various physical and biological quantities for the reference experiment.

Next, the nutrient concentration has to be calculated. As a first step, we neglect the contributions in 2a that depend on the phytoplankton population density $P$ and focus on the nitrate and nitrite concentrations as those are growth-limiting in the lower estuarine regions. From observations it follows that $N$ is approximately constant over depth (see Sin et al. [1999], figure 7). Following McCarthy
[1993], an expression for the leading order longitudinal distribution of \( N \) can be obtained by making an expansion in the small parameter \( \varepsilon \). Using this information and the light intensity, the specific growth rate \( \mu (N, I) \) and hence the phytoplankton density can be calculated.

**Results and Discussion**

In the experiment discussed here, results are given using parameter settings representative for the York river. A small river outflow is prescribed (10m\(^3\)/s). In figure 2(a) the prescribed horizontal density profile and the vertical diffusivity parameter is shown. Due to a stable stratification in the neighbourhood of the transition from salty seawater to fresh water, the vertical mixing is locally reduced. In figure 2(b) the along-estuary profile of \( N \) is shown. At the riverine side, both the nutrient concentration and the flux of nutrients is prescribed. In the seaward part of the estuary the nutrient concentration does not vary anymore. Figure 2(c) shows the time-averaged concentration profiles in the estuary. Due to the presence of an along-channel varying vertical diffusivity, a (weak) second ETM is observed near the entrance of the estuary. Note that the concentrations in this second ETM are much smaller than in the main one. Combining the information shown in these two figures, the specific growth rate can be calculated. The spatial structure of \( \mu \) is shown in figure 2(d). From this figure it can be concluded that the growth rate is light limited near the main ETM and nutrient limited in the more seaward part of the estuary. As a next step, the phytoplankton equation (2b) has to be solved. Since the boundary conditions are homogeneous, no trivial steady state solution exists. Hence, phytoplankton behaviour will be time dependent. This corresponds well with the observations of the episodic character of the algal blooms. Results of this temporal behaviour will be shown and discussed.

**References**


