Tides in Equilibrium River Valley Estuaries

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ABSTRACT

Introduction

In introductory oceanography, the physics of estuarine tides is often presented as the superposition of an incident and reflected wave in a constant-width channel. In reality, tides in most river valley estuaries are more accurately and easily understood as unidirectional propagating waves that do not significantly reflect. In contrast to the partial standing wave paradigm, tidal height and velocity amplitude along many river valley estuaries are determined at lowest order by a local competition between dissipation by friction and convergence associated with an along-channel decrease in cross-sectional area (Jay 1991; Friedrichs and Aubrey, 1994). This description is key to understanding the rate at which tidal phase propagates landward through estuaries, to interpreting the local timing of high tide relative to slack water, and to understanding along-estuary variations in the magnitude of observed tidal range and velocity. In such estuaries, a morphodynamically stable (i.e., “equilibrium”) channel is characterized by a nearly uniform amplitude for tidal elevation along the length of the estuary (Prandle, 2004). If average channel depth and bottom drag along the estuary are relatively uniform, then the maximum tidal velocity, minimum channel depth and maximum rate of width convergence are all uniquely specified for a given tidal range. In this paper, the predictions of equilibrium estuary tidal theory are compared favorably to river valley estuaries found along the mid-Atlantic coastal plain of the U.S.

Results

Several river valley estuaries along the U.S. mid-Atlantic coast are characterized by channel cross-sectional areas that converge exponentially landward, whereas tidal range and cross-sectionally averaged tidal velocity remain relatively uniform with distance along-channel (Figure 1). This suggests that these systems are close to the “equilibrium” case where the tendency for friction to damp tidal amplitude is almost exactly balanced by the tendency of convergence to increase tidal amplitude.

Based on the observed geometry and tidal dynamics of coastal plain estuaries along the U.S. mid-Atlantic, an idealized river valley estuary is assumed at lowest order to have an exponentially decreasing cross-sectional area, to be of relatively constant depth, to be characterized by a spatially uniform amplitude for tidal elevation and velocity, and to exhibit a constant relative phase between the two. Then starting with the linearized, cross-sectionally averaged equations for conservation of mass and momentum, it can be shown that

\[ L = h \frac{(gh)^{1/2}}{(c_d U)} \]

Where \( L \) is the e-folding length for the along-channel convergence of cross-sectional area, \( h \) is channel depth, \( g \) is the acceleration of gravity, \( c_d \) is the bottom drag coefficient, and \( U \) is the amplitude of tidal velocity. Using this relation to eliminate \( L \) from the governing equations, it can be further shown that the following relationship must hold among channel depth, tidal frequency \((\omega)\), tidal velocity and the amplitude of tidal elevation \((a)\):

\[ h^2 - (g a^2 U^2) - g c_d U^2 \omega^2 = 0 \]

The above relationship for channel depth has a real solution only for tidal velocity amplitudes less than the following maximum value \((U_{\text{max}})\):
\[ U_{\text{max}} = 2^{1/3} \left( g \cdot \omega^{-1} c_d a^2 \right)^{1/3} \]

\( U_{\text{max}} \) varies among equilibrium estuaries mainly as a function of tidal range (since the drag coefficient and tidal period are assumed to be system independent). This constraint on maximum tidal velocity then places analogous constraints on minimum channel depth \( (h_{\text{min}}) \) and on the minimum e-folding length-scale for channel width convergence \( (L_{\text{min}}) \), again mainly as a function of tidal range:

\[ h_{\text{min}} = \frac{1}{2} \left( g^{1/2} \cdot \omega_c^{-1} c_d a \right)^{2/3} \]

\[ L_{\text{min}} = 2^{7/6} \left( g^2 \cdot \omega_c^{-4} c_d a \right)^{1/3} \]

Table 1 displays observed values for tidal range \((2a)\), \(U\), \(h\) and \(L\) for the Rappahannock, James, Potomac and Delaware estuaries along with their theoretical values for \(U_{\text{max}}, h_{\text{min}}, L_{\text{min}}\) (assuming \(c_d = 0.0025\)). In each case, the observed cross-sectionally averaged tidal velocity is quite close to the theoretical maximum value. The observed depths and width convergence rates tend to be somewhat larger than the theoretical minimums values, suggesting these estuaries may still be in the process of infilling toward their final equilibrium shapes.

**Conclusions**

The major conclusions of this work include the finding that equilibrium tidal velocity in river estuaries may be determined largely from the tidal range without knowing channel depth or the rate of along channel width convergence apriori. Furthermore, the resulting equilibrium tidal velocity is unrelated to the specific sediment properties of the estuary. This implies that the sediment types that are trapped in such estuaries are ultimately the result of the equilibrium estuary shape rather than the other way around. The theoretical constraints on equilibrium depth and convergence rate are also consistent with observations, in that the observed values are relatively close to and always no smaller than their theoretical minimums.

**References**


Figure 1. Observed along-channel variation in channel cross-sectional area, tidal range and the amplitude of cross-sectionally averaged tidal velocity along four U.S. mid-Atlantic river valley estuaries.

Table 1. Tidal range and observed and predicted velocity, depth and e-folding lengths for four U.S. mid-Atlantic coastal river valley estuaries.

<table>
<thead>
<tr>
<th>Estuary</th>
<th>Tidal range (m)</th>
<th>U (m/s)</th>
<th>U_{max} (m/s)</th>
<th>h (m)</th>
<th>h_{min} (m)</th>
<th>L (km)</th>
<th>L_{min} (km)</th>
</tr>
</thead>
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<tr>
<td>Rappah.</td>
<td>0.7</td>
<td>0.3</td>
<td>0.3</td>
<td>4</td>
<td>4</td>
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<td>26</td>
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<td>0.4</td>
<td>5</td>
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<td>29</td>
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<td>0.6</td>
<td>6</td>
<td>6</td>
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<td>35</td>
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