Gravity-driven sediment transport on the continental shelf: implications for equilibrium profiles near river mouths

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Abstract

An analytical model is developed for equilibrium bathymetric profiles off river mouths associated with the shoreward, convex upward portion of subaqueous deltas and clinoforms. The model builds on recent field results demonstrating that gravity-driven flux of suspended mud is important on shelves provided that wave-induced suspension of sediment supports the requisite turbid hyperpycnal layer. Because the maximum sediment load is determined by the critical Richardson number, the results are independent of the properties of the suspended mud or the bed. The model assumes the equilibrium state to represent a balance between the supply of sediment by a river at the coast and the downslope bypassing of sediment to deep water within wave-supported turbid near-bed layers. Progressive seaward increases in bed slope across the convex shelf profile allow the attenuation of wave agitation with depth to be compensated for by a downslope increase in the contribution of gravity. The model is consistent with shelf profiles off the mouths of the Eel (California), Ganges-Brahmaputra (Bangladesh), Waiapu (New Zealand), Po (Italy), and Rhone (France) Rivers. The equilibrium profile is predicted to be a function of wave climate and riverine sediment supply only, with deeper and broader profiles associated with decreasing sediment supply, increasing wave height and/or increasing wave period.

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1. Introduction

A fundamental underpinning of most models of coastal morphodynamic behavior is the notion that dynamic equilibrium states exist and that changes, either erosional or progradational, occur in ways that minimize departures from those states. Generally, the equilibrium is defined in terms of minimization of gradients in net sediment flux over the area of interest. For over a century, it has been understood that concave upward profiles typically front wave-dominated beaches that lack regular input of new sediment, whereas river deltas and river mouth regions that regularly receive new sediment exhibit convex upward profiles (e.g., Wright, 1995). In the geological record, the latter situation is associated with the landward, convex upward portion of clinoforms, the
sigmoid-shaped surfaces that are the building blocks of prograding stratigraphic sequences (Pirmez et al., 1998). In this paper, we present an equilibrium model for such convex upward profiles in the presence of wave-supported, gravity-driven sediment transport.

In the case of the classic concave upward nearshore profile of equilibrium, the prevailing shape has typically been considered to express and accommodate a uniform rate of wave energy dissipation, either per unit volume of water column or per unit area of bed (Dean and Dalrymple, 2002). When the dissipation rate is not uniform and a disequilibrium condition exists, it is assumed, in such models, that profile shape readjustment takes place by sediment being diffused from regions of higher dissipation to regions of lesser dissipation. In such conceptualizations, it is diffusive processes and not advective processes, such as gravity-driven or current-driven transport that dominate. However, Dean and Dalrymple (2002) show that inclusion of advective gravitational effects yields a flatter profile than would be predicted from uniform dissipation alone.

Simple models for equilibrium clinoform surfaces have also emphasized diffusive processes (e.g., Kenyon and Turcotte, 1985; Syvitski et al., 1988). As pointed out by Pirmez et al. (1998), a shortcoming of slope-driven diffusion approaches is that only concave-up profiles are produced, which leaves such models unable to explain the landward, convex portion of the clinoform. The resulting solution makes the clinoform roll-over point (point of sharpest change in gradient) unrealistically coincident with the shoreline, despite field observations that the roll-over point for clinoforms and subaqueous deltas is located away from the shoreline and that the depth of the roll-over increases with hydrodynamic energy (Pirmez et al., 1998).

Pirmez et al. (1998) produced a convex upward shoreward component for clinoforms off river mouths by applying an advective model that assumes deposition is inversely proportional to bed stress. A constant across-shelf volume flux of water was used to represent a residual advection velocity into the basin, and bed stress was related to across-shelf volume flux divided by local depth. As the clinoform approached a stably prograding profile, the shoreward convex component evolved to a predominantly bypassing, higher stress region with rapid deposition beginning seaward of the roll-over. However, the physics that might correlate riverine discharge to across-shelf advective stress were not explicitly addressed. In simulating clinoforms with a more physics-based model, O’Grady and Syvitski (2001) used slope failure to trigger advective transport by turbidity currents. However, the turbidity currents were auto-suspending and triggered by slope failure. Thus, they mainly occurred seaward of the clinoform roll-over.

Most continental shelves slope too gently to support auto-suspension in turbid hyperpycnal flows. For that reason, the role of gravity-driven turbidity flows in advecting sediment across the shoreward portions of subaqueous deltas and clinoforms was previously considered unimportant, except in relatively rare cases where rivers discharged hyperpycnal sediment concentrations directly into coastal seas (e.g., Wright et al., 1988). However, recent developments in field instrumentation capable of resolving near-bed flows and sediment concentrations, and the applications of that technology in the context of the U.S. Office of Naval Research (ONR) STRATAFORM program on the northern California shelf off the Eel River have changed that thinking. Specifically, several investigators have reported strong evidence that hyperpycnal layers do not require auto-suspension for sustenance, but can be created and supported by wave- and current-induced resuspension within relatively thin near-bed layers (Ogston et al., 2000; Traykovski et al., 2000; Wright et al., 2001, 2002; Scully et al., 2002, 2003). As these layers move downslope under the influence of gravity, they may deposit sediment in response to decreases in bottom orbital velocities, bed slope or both. Alternatively, if the slope is sufficiently steep and/or the wave–current-induced resuspension is sufficiently intense to prevent deposition, the downslope sediment flux may balance the supply of new sediment from river mouths or other upslope sources, and a local and time varying state of net bypassing may prevail.

In this paper, we introduce a model for the development of shelf profiles that maintain dynamic equilibrium through a relatively simple suite of processes involving river supply of fine sediment, wave-supported negatively buoyant turbid layers, and gravity-driven downslope transport of those layers.
This model yields a profile shape that appears to be consistent with the shoreward, convex upward portion of subaqueous deltas or clinoforms off numerous high-yield river mouths. We have selected five examples of shelves and/or subaqueous deltas off the following rivers to illustrate the model presented here: the Eel River, California, USA; the Waiau River, New Zealand; the Ganges-Brahmaputra River system, Bangladesh; the Po River, Italy; and the Rhone River, France.

2. Theoretical development

The time-averaged, depth-integrated momentum balance for a wave-supported sediment gravity flow, within which negative buoyancy results from sediment suspension by waves, is given approximately by a balance between the downslope pressure gradient induced by the turbid layer and opposing mean bottom stress (Fig. 1):

\[ a \sin \theta = \rho_s |u_g| \]  

(1)

In Eq. (1), \( a \) is the sine of the bed slope, \( g \) is the acceleration of gravity, \( \rho_s \) is the density of siliceous sediment and \( s \) is its submerged weight relative to sea water, \( C \) is the depth-integrated mass concentration of suspended sediment within the wave boundary layer, and \( c_d \) is the bottom drag coefficient (Wright et al., 2001; Scully et al., 2002). The key velocities associated with Eq. (1) are the downslope velocity of the gravity current \( (u_g) \), the absolute amplitude of the instantaneous velocity, \( |u| = (u_w^2 + u_g^2)^{1/2} \), all evaluated near the top of the wave boundary layer. The form of the right-hand side of Eq. (1) arises from the widely accepted quadratic formulation for wave-averaged stress at the base of a bottom boundary layer in the presence of both wave orbital velocity and a mean current (e.g., Grant and Madsen, 1979; Feddersen et al., 2000).

A second key relation is the Richardson number for the wave boundary layer:

\[ Ri = \frac{gsC \rho_s^{-1} |u|^{-2}}{c_d} \]  

(2)

which compares the relative influence of buoyancy and shear on the generation of turbulence by shear instabilities within the wave boundary layer. For wave-supported gravity flows on the Eel River shelf, Wright et al. (2001) and Scully et al. (2002) showed that observed transport and deposition rates could be explained by Eqs. (1) and (2) in combination with a feedback mechanism which maintains \( Ri \) in Eq. (2) near its critical value of \( Ri_c = 1/4 \). For \( Ri < 1/4 \), turbulence associated with intense shear instabilities suspends additional sediment, increasing \( C \) and \( Ri \), while for \( Ri > 1/4 \), decreased generation of shear instabilities reduces turbulence and causes sediment to settle. Because the maximum sediment load is determined by the critical Richardson number, this approach for predicting sediment concentration is not dependent on detailed sediment or bed properties. The only requirements regarding the sediment are that a sufficient supply of easily suspended material is available to produce critical stratification, and that

\[ C = \int_0^\delta c \, dz \]

\[ |u| = (u_w^2 + u_g^2)^{1/2} \]

Fig. 1. Schematic diagram of a sediment-induced gravity current of depth-integrated concentration \( C \), trapped within a wave boundary layer of thickness \( \delta \), and moving across a continental shelf of bed slope \( \theta \). The velocity scale \( |u| \) accounts for the contribution to quadratic friction of both wave orbital velocity \( (u_w) \) and the speed of the gravity current \( (u_g) \).
its fall velocity is small enough that it stratifies the full thickness of the wave boundary layer.

Combining Eqs. (1) and (2) and the definition of \(|u|\) to eliminate \(|u|\) and solve for \(u_g\) and \(C\) then yields the following analytical solutions for the critically stratified case of \(R_i = R_i_c\):

\[
 u_g = u_w(xR_i_c c_d^{-1}) \left\{ 1 - \left( xR_i_c c_d^{-1} \right)^2 \right\}^{-1/2} \tag{3}
\]

\[
 C = R_i c_w u_w^2 g^{-1/2} \left\{ 1 - \left( xR_i_c c_d^{-1} \right)^2 \right\}^{-1} \tag{4}
\]

Eqs. (3) and (4) indicate that across-shelf sediment flux due to wave-supported gravity flows \((u_g C)\) increases with wave orbital velocity and bed slope. Because bottom orbital velocity decreases with depth, sediment flux will decrease with greater water depth if bed slope remains constant. Eqs. (3) and (4) also indicate that gravity flows depend on wave support for existence only when \(x < c_d R_i_c^{-1}\). For \(x \geq c_d R_i_c^{-1}\), the bed is sufficiently steep for critically stratified gravity flows to auto-suspend as they accelerate down-slope (Wright et al., 2001). Of course, in such cases, waves may still assist the auto-suspension process in maintaining negative buoyancy. For \(x < c_d R_i_c^{-1}\) (which typifies most shelves), turbulent gravity flows cannot be maintained without an external source of bed shear. This slope criterion explains why auto-suspending turbidity currents occur more often on the continental slope than on shelves.

Off river mouths where the shelf is strongly convex upward and \(x\) increases progressively with depth, \(h\), a critical depth may be reached at which a critical slope, \(z_c = c_d R_i_c^{-1}\) is attained. Beyond this depth, gravity flows may be self-sustaining via auto-suspension (Wright et al., 2001). As \(u_g\) exceeds \(u_w\), it is also unlikely that the thin hyperpycnal layer will remain trapped within the wave boundary layer. Rather, the gravity flow will grow in thickness as a developing current boundary layer. In general, the critical slope, \(z_c = c_d R_i_c^{-1}\) is in the range 0.01–0.02 (with \(c_d \approx 0.003–0.005\)); hence in subsequent discussion, we designate the critical depth at which this slope is approached as \(h_{0.01} = h(z=0.01)\). Much beyond \(h_{0.01}\), theory based only on wave-supported gravity flows cannot be used to confidently predict equilibrium morphology.

The convex upward portion of a subaqueous delta or clinoform subject to wave-supported gravity flows will be at equilibrium if there are no across-shelf gradients in gravity driven flux, and the available river sediment supply matches the capacity of wave-supported gravity flows to remove sediment. In other words, the equilibrium profile at depths \(\leq h_{0.01}\) requires

\[
 u_g C = Q_t \tag{5}
\]

where \(Q_t\) is the supply of riverine sediment per unit distance along-shelf. Applying linear wave theory,

\[
 u_w = \omega (H/2)(\sinh kh)^{-1} \tag{6}
\]

where \(H\) is wave height, \(\omega\) is radian frequency, \(h\) is water depth, and \(k\) is wave number given by the dispersion relation

\[
 k = \omega^2 (g \tanh kh)^{-1} \tag{7}
\]

Combining Eqs. (3)–(6) to eliminate \(u_g\), \(C\) and \(u_w\) gives the following relation for equilibrium bathymetric slope:

\[
 z \left\{ 1 - \left( x R_i_c c_d^{-1} \right)^2 \right\}^{-3/2} = 8(\omega H)^{-3}(\sinh kh)^3 Q_t s g c_d R_i_c^{-1} \rho_s^{-1} \tag{8}
\]

Eq. (8) predicts that the slope of an equilibrium profile dominated by wave-supported gravity flows increases with greater water depth and sediment supply, and decreases with increasing wave height and wave period (via \(k\)). Equilibrium slope increases with water depth to compensate for the effect of decreasing bottom orbital velocity. Slope increases with sediment supply simply because a greater slope is required to transport a larger sediment supply offshore. Slope decreases with increased wave period because a greater period decreases the decay of \(u_w\) with depth. Finally, equilibrium slope decreases with increased wave height to compensate for the effect of greater \(u_w\).

Fig. 2 displays equilibrium profiles for realistic ranges of the above parameters. These plots clearly show that the equilibrium profile is predicted to be deeper and broader with decreasing sediment supply, increasing wave height and/or increasing wave period. Note that the \(x\)-offset for each profile in Fig. 2 is
arbitrary, since the solution for across-shelf distance involves an integration constant and does not imply an absolute distance from the shoreline. For the asymptote of shallow water waves with $u_w \gg u_g$, Eq. (8) reduces to

$$
\alpha = \left(8c_{d}Ri_c^{-2} \rho_s^{-1} s^{-1/2}\right)Q_r H^{-3}h^{3/2}
$$

(9)

This asymptote conveniently isolates the constants $c_d$, $Ri_c$, $\rho_s$ and $g$, such that a very simple relation holds among the morphodynamic variables for large orbital velocity shallow water waves, namely

$$
\alpha \sim h^{3/2} Q_r/H^3
$$

(10)

It is worth re-emphasizing here that the equilibrium solutions in Fig. 2 specifically apply to the convex upward, landward portion of potentially sigmoid shelf profiles. The equilibrium model presented here (which assumes the profile is fixed in space and does not consider effects of changing sea level) cannot explain
as an equilibrium form the concave upward profile that is characteristic of the deeper, offshore portion of subaqueous deltas and clinoforms. An interpretation consistent with the dynamics of wave-supported sediment gravity flows might be that the landward, convex portions of such profiles completely bypass and are in stationary equilibrium, while the concave portions are depositional and in an evolving disequilibrium associated with deposition. However, the landward portions of deltas and clinoforms are rarely morphodynamically static, and in geologic time, the entire sigmoid can represent a stable form that progrades as a unit (Pirmez et al., 1998). Practically speaking, the equilibrium state characterized by the above equations may still reasonably characterize the landward portions of real bathymetric profiles, which in an event-averaged sense bypass the clear majority of riverine sediment into deeper water. Over geological time, it only requires local deposition of a small minority of total sediment discharge to cause a convex upward shape that is reasonably described by equilibrium assumptions to vertically accrete or horizontally prograde.

3. Application to Eel River shelf

As a result of the STRATAFORM investigations, reasonable constraints can be placed on the Eel River
shelf environment (Fig. 3) in terms of waves, bathymetry, and riverine sediment supply during major events likely to dominate equilibrium morphology. Most significantly, bathymetry and sediment supply are well documented as functions of distance along-shelf. Fine sediment supply to the shelf is biased to the north of the river mouth because the peak stages of Eel River floods are consistently associated with strong winds from the south. As the plume loses sediment through gravitational settling, the plume’s concentration and resulting sediment loss (deposition) rate decrease exponentially to the north (Geyer et al., 2000). This provides an opportunity to test the ability of Eq. (8) to predict equilibrium bed slope as a function of both depth and sediment supply without the confounding influence of region-specific wave properties.

In mks units, the dimensional values appropriate to Eq. (8) are $g=9.8 \text{ m}^2/\text{s}$ and, for siliceous sediment, $\rho_s=2600 \text{ kg/m}^3$. The appropriate dimensionless parameters are $R_i=0.25$, $c_d=0.003–0.005$ and, for siliceous sediment, $s=1.6$. The remaining inputs are chosen to represent the type of major events likely to dominate across-shelf sediment transport and deposition over morphologically relevant time scales. For application to the bathymetric profile of the Eel River shelf, discharge and waves are based on characteristic properties of the January 1995 and January 1997 floods as documented by Wheatcroft and Borgeld (2000), with a duration of ~15 days and an event-integrated fine-grained suspended sediment load of ~30×10^9 kg.

As noted by Scully et al. (2002), the slope of the mid- to outer Eel River shelf is markedly steeper close to the river mouth than further north in the vicinity of the mud depocenter. Fig. 4 displays observed and predicted shelf slopes for these two regions, located 0–15 km (○) and 20–35 km (×) north of the Eel mouth. For Fig. 4, observed slope is the absolute depth gradient calculated from NOS bathymetry using a second-order centered difference, with an across-shelf grid spacing of 400 m and an along-shelf grid spacing of 1000 m. Following Scully et al. (2002), $Q_r$ (across-shelf load of fine riverine sediment available per unit length along-shelf) is assumed to diminish exponentially with distance to the north of the river mouth, with an e-folding length of 20 km. The specific values of $Q_r$ applied in Fig. 4 (0.81 and 0.30 kg/m/s) are then the average loads over the relevant along-shelf segment.

The solution for the equilibrium slope given by Eq. (8) is highly sensitive to wave height and period and
moderately sensitive to the drag coefficient. The near-bed peak period for orbital velocity during storms on the Eel shelf is documented by Wright et al. (1999) and Traykovski et al. (2000) to be about 14 s. The cases displayed in Fig. 2 apply \( c_d = 0.004 \) and an rms wave height of \( H = 3 \) m (equivalent to a significant wave height of 4.2 m, where \( H_{\text{sig}} = 2^{1/2} H \)). The chosen wave height is reasonably consistent with Wheatcroft and Borgeld (2000), who document typical significant wave heights of 4 to 5 m during the January 1995 and January 1997 floods.

Since the wave height chosen in fitting the equilibrium model to the Eel Shelf is somewhat tuned, more objective tests for the model are: (i) its ability to simultaneously reproduce observed shelf slope both near and far from the river mouth as a function of varying spatially variable sediment supply, and (ii) the relationship between spatially varying deviations from Eq. (8) and zones of observed bypassing and deposition. Fig. 3 indicates that on the mid-shelf \((h \approx 70 \) m\), the shelf slopes near the river mouth \((0–15 \) km\) and near the observed mid-shelf depocenter \((20–35 \) km\) are indeed both simultaneously close to equilibrium values for \(H = 3 \) m. The equilibrium slope of the mid-shelf nearer the river mouth is steeper because a steeper slope is required there to move a significantly larger supply of riverine sediment offshore. For both sites, the slope in the vicinity of the mid-shelf also increases offshore in a manner more-or-less consistent with Eq. (8). At equilibrium, shelf slope must increase offshore so that \( h_g \) increases sufficiently to compensate for decreased sediment concentration associated with reduced bottom orbital velocity.

Along both profiles, the slope of the inner shelf is markedly steeper than the predicted equilibrium, suggesting that fine sediment bypassing is favored close to shore. Thus, the sandy and concave upward bathymetric profile of the inner shelf off the Eel River cannot be explained by wave-supported gravity flows as described by Eqs. (1) and (2). This is not too surprising, since the settling velocity of sand is too high to allow it to be maintained in suspension at sufficient concentrations to critically stratify the full thickness of the wave boundary over extended periods. Nonetheless, wave-induced gravity flows may still play an important role in the transport of sand across the inner shelf. For example, Wright et al. (2002) document short-lived periods of accelerated seaward transport of sand on the inner shelf off Duck, NC, following higher groups of storm waves, when sediment-induced \( Ri \) briefly exceeds 1/4. A concave upward inner shelf may be conceptually consistent with down-slope sandy gravity flows balanced by onshore-directed wave asymmetry.

On the mid-shelf off the Eel, deposition is marginally possible once the observed slopes approach the equilibrium lines predicted by Eq. (8). The first depths at which this occurs for \(H = 3 \) m are for \( h \) between 45 and 65 m, consistent with the observed location of the sand–mud transition off the Eel River (Fig. 4). Moving further offshore, the observed slope near the river mouth remains centered on the equilibrium line before exceeding equilibrium once more at depths

<table>
<thead>
<tr>
<th>Site</th>
<th>( h ) (m) slope=0.01 observed</th>
<th>( H ) rms (m) during storm</th>
<th>( T ) (s) at ( h )</th>
<th>Annual load (10^5 kg/year)</th>
<th>Drainage area, ( A ) (1000 km^2)</th>
<th>Flood duration (days)= 5.85 ( A^{0.44} )</th>
<th>Along-shelf spread (km)</th>
<th>( Q_r ) (kg/m/s)</th>
<th>( h ) (m) slope=0.01 (Eq. (8))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eel, 20–35 km</td>
<td>100</td>
<td>3.0</td>
<td>14.0</td>
<td>30^a</td>
<td>8.6</td>
<td>15</td>
<td>not applicable</td>
<td>0.30</td>
<td>96</td>
</tr>
<tr>
<td>Eel, 0–15 km</td>
<td>80</td>
<td>3.0</td>
<td>14.0</td>
<td>30^a</td>
<td>8.6</td>
<td>15</td>
<td>not applicable</td>
<td>0.81</td>
<td>79</td>
</tr>
<tr>
<td>Ganges-Brahm.</td>
<td>40</td>
<td>1.7</td>
<td>9.2</td>
<td>1060</td>
<td>1650</td>
<td>150</td>
<td>200</td>
<td>0.41</td>
<td>35</td>
</tr>
<tr>
<td>Waiapu</td>
<td>30</td>
<td>2.5</td>
<td>10.3</td>
<td>35</td>
<td>1.7</td>
<td>7</td>
<td>10</td>
<td>5.5</td>
<td>28</td>
</tr>
<tr>
<td>Po</td>
<td>13</td>
<td>0.8</td>
<td>6.1</td>
<td>15</td>
<td>74</td>
<td>39</td>
<td>20</td>
<td>0.23</td>
<td>14</td>
</tr>
<tr>
<td>Rhone</td>
<td>4</td>
<td>1.4</td>
<td>7.6</td>
<td>59</td>
<td>98</td>
<td>44</td>
<td>4</td>
<td>3.9</td>
<td>13</td>
</tr>
</tbody>
</table>

^a January 1995, 1997 flood event.
greater than 85 m. In contrast, the observed slope near
the depocenter (20–35 km north) tends to be centered
slightly below equilibrium until 120 m depth. The
existence of the depocenter here suggests that the
slope must be less than the equilibrium slope;
otherwise, net deposition would not occur. Closer
to the river mouth, net deposition is not strongly
favored on the mid-shelf, which is more precisely consistent
with a long-term equilibrium. Approaching the shelf
break, however, observed slope exceeds equilibrium
\( \gamma > 0.016 \) at both locations, favoring
gravity-driven bypassing once more.

Enhanced bypassing of sediment on the outer shelf
is consistent with the observed offshore limit of the
active mud deposit (Wheatcroft and Borgeld, 2000),
but it is not consistent with an equilibrium morphology
for the shelf break as predicted by wave-
supported gravity flows. This suggests that equili-

brium morphology at greater depths and slopes off the
Eel River is controlled by other processes, such as
tectonically induced slope failure and auto-suspending
turbidity currents. Gravity flows are also less likely to
be confined to the wave boundary layer once the
strength of the mean current approaches or exceeds
the strength of the bottom orbital velocity.

4. Application to other shelves

As an extension to the STRATAFORM program,
the joint ONR-European Union EuroSTRATA-
FORM project is presently applying techniques
and understanding of sediment dynamics developed
through study of the Eel margin to shelves off the
Po River in Italy and the Rhone River in France.
Like the Eel shelf, these two sites are also
characterized by active subaqueous deltas, although
with \( h_{0.01} \) in much shallower water \((<15 \text{ m})\). The
depth at which the slope of each profile reaches
0.01 is given in Table 1. Two riverine shelves with
active shelf subaqueous deltas/clinoforms at inter-
mediate depths \( (h_{0.01} \sim 20 \text{ to } 40 \text{ m}) \) are off the
Waiapu River in New Zealand and off the Ganges-
Brahmaputra River system in Bangladesh. Fig. 5
displays bathymetry for these four riverine shelves
in order from deepest to shallowest \( h_{0.01} \). The
profile lines superimposed on the maps indicate the
portions of the subaqueous deltas which are convex
upward and thus potentially consistent with our
equilibrium model. Fig. 6 provides plots of depth
versus distance offshore for these four systems plus
the two sites along the Eel River shelf.

In an effort to be objective in defining character-

istic waves for these six shelves, a 30-year global
wave hindcast (Oceanweather, 2003) was used to
provide a consistent measure of wave height statistics
for the various sites. The version of the Global
Reanalysis of Ocean Waves (GROW) hindcast used
here is a more highly resolved iteration \((0.625^\circ \text{ in latitude by } 1.25^\circ \text{ in longitude})\) of the initial GROW
hindcast \((1.25^\circ \times 2.5^\circ)\) described by Cox and Swail
(2001). The 90th percentile rms wave height \( (H_{90}) \)
hindcast by GROW for the Eel shelf is 2.83 m, which
compares favorably with \( H = 3.0 \text{ m} \) assumed for
storms in Fig. 3. The ratio of \( H \) to \( H_{90} \) found to
apply well to the Eel River shelf is used as a
calibration for the other five shelves, and the
characteristic wave height for the morphology of each
of the remaining shelves is set to \((3/2.83)\) times the
particular \( H_{90} \) calculated by GROW offshore of each
other river mouth (Table 1).

The above approach for estimating the character-

istic wave height relevant to subaqueous delta
morphology is admittedly limited in several respects.
For example, the GROW hindcast models only
deeper wave waves, and the waves which impinge
on the subaqueous deltas are affected by the finite depth
of the shelf. Furthermore, it is not clear that \( 1.06 \times \)
\( H_{90} \) should be the most relevant wave height for the
other shelves even if it were an appropriate choice for
the Eel Shelf. Storms and floods off the Eel River are
more highly coupled in time than similar events off
larger rivers because of the relatively small size of the
Eel River drainage basin (Wheatcroft, 2000). This
suggests that \( H_{90} \) may overestimate the characteristic
wave height during floods offshore of larger drainage
basins. Nonetheless, \( 1.06 \times H_{90} \) value
previously determined by GROW provides an esti-
mate that is simple to calculate and probably more
objective and consistent than tuning the characteristic
wave height for each shelf individually. The main
aim here is not to accurately predict the clinoform
profile from equilibrium theory but rather to explore
whether or not the equilibrium theory might provide
a possible and reasonable explanation for the
observed bathymetry.
Fig. 5. Location map and bathymetry for shelves off (a) the Brahmaputra River system in Bangladesh (modified from Kuehl et al., 1997), (b) the Waiapu River, New Zealand (modified from NIWA, 1996), (c) the Po River, Italy (bathymetric data from Correggiari et al., 2001, shoreline data from Soluri and Woodson, 1990), and (d) the Rhone River, France (modified from Berné et al., 2002). The profile lines indicate the well-resolved, convex upward portion of the subaqueous deltas plotted in Fig. 6.
For the Eel shelf, the peak period for bottom orbital velocity over the convex portion of the clinoform was directly observed by tripods to be about 14 s during storms (Wright et al., 1999; Traykovski et al., 2000). For cases other than the Eel, peak surface wave period in seconds was set to $5.0H_{\text{sig}}^{1/2}$, as given by the Pierson–Moskowitz spectrum for a fully developed sea in deep water under steady winds (Carter, 1982). Individual frequency components of the spectrum were then decayed according to linear wave theory to predict the peak period associated with bottom orbital velocity at the depth, $h_{0.01}$, where the slope of the subaqueous delta/clinoform was observed to reach 0.01 (Table 1). The Jonswap spectrum (Carter, 1982), which has been shown to be more appropriate over broad shelves, was also considered. The Pierson–Moskowitz spectrum was used instead for two reasons. First, the Pierson–Moskowitz spectrum is more consistent with the approach used in the GROW deepwater wave model, and second, the Pierson–Moskowitz spectrum predicted peak periods for bottom orbital velocity which were more consistent with those applied to the Eel shelf in Fig. 3.

For the rivers other than the Eel, the instantaneous load during a characteristic flood was calculated as the annual load divided by the characteristic annual flood duration over which the vast majority of sediment is delivered to the ocean. The annual suspended load of each river was provided by Milliman and Farnsworth (2005) (Table 1). The characteristic duration for the morphodynamically relevant flood, known from STRATAFORM to be about 15 days for the Eel (Wheatcroft and Borgeld, 2000), was scaled to longer or shorter durations based on the size of the
drainage area of each river relative to the Eel. At the upper extreme, the flood season in terms of high sediment load lasts about 150 days for the Ganges-Brahmaputra River system (Islam et al., 2002). Despite the large size of the Ganges-Brahmaputra River system, very little sediment is discharged outside this period. The annual flood durations for the remaining systems were estimated from a log–log interpretation/extrapolation between drainage area and flood duration as defined by the relative extremes of the Eel and Ganges-Brahmaputra, with drainage area taken from Milliman and Farnsworth (2005) (Table 1). Again, the aim here is not to precisely predict observed morphology, but rather to investigate the potential range of real world applicability. A log–log interpolation/extrapolation based around the Eel and Ganges-Brahmaputra provides reasonable first-order estimates of characteristic flood durations without the potential subjectivity of tuning each case individually.

In order to estimate \( Q_r \) in units of kg/m/s, it was necessary to specify an along-shelf distance off each river mouth over which the sediment load was spread. For each river other than the Eel (for which the affect of along-shelf spreading is taken directly from Scully et al., 2002), the along-shelf spreading distance is based on the along-shelf scale of the subaqueous delta/clinoform and/or spacing of multiple river mouths as inferred from the bathymetry and shoreline displayed in Fig. 5. To one significant digit, the along-shelf length scales based on these maps are about 200, 20, 10, and 4 km for the Ganges-Brahmaputra, Po, Waiapu and Rhone, respectively. For cases other than the Eel, \( Q_r \) was finally calculated as annual load divided by flood duration divided by along shelf spreading distance (Table 1).

Eq. (10) indicates that for the asymptote of energetic waves in shallow water, the depth at which equilibrium profile slope reaches a given value such as 0.01 should be a function only of \( H^3/Q_r \) (since the other variables are constant from site to site). Fig. 7(a) displays the observed depths for \( a=0.01 \) versus observed values of \( H^3/Q_r \). A more-or-less monotonic relationship is seen between \( h(a=0.01) \) and \( H^3/Q_r \), which is consistent with the result that \( H^3/Q_r \) is a useful parameter for predicting the relative depth of

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**Fig. 5 (continued)**.
subaqueous deltas/clinoforms. However, the observed values of $h_{0.01}$ are consistently lower than the actual equilibrium depths predicted for shallow water by Eq. (9) (also shown on Fig. 7(a)). This is because $h_{0.01}$ is too deep off most of the rivers to quantitatively apply the shallow water asymptote. The limited wave periods at most of the sites causes rapid depth attenuation of bottom orbital velocity with $h$, requiring a steeper shelf at equilibrium to compensate for decreasing $u_w$.

Fig. 7(b) compares observed $h_{0.01}$ to the values predicted by Eq. (8), which does not assume shallow water waves. For most of the sites, the observed depth of the subaqueous delta/clinoform is close to that predicted by the equilibrium theory if one accounts for both $H^3/Q_r$ and $T$. This result suggests that equilibrium bypassing of wave-supported gravity currents is generally consistent with the observed slope and depth of the convex portion of subaqueous deltas and clinoforms subject to a wide range of wave forcing and sediment loading. Because Fig. 7(a) and (b) display monotonic relationships, with and without consideration of $T$, it is further evident that the effects of $H^3/Q_r$ and $T$ tend to be correlated, i.e., locations with large or small $H^3/Q_r$ tend to experience long- or short-wave periods, respectively. This is why none of the theoretical profiles in Fig. 2 encompass the extremely shallow or deep observed profiles in Fig. 6. Fig. 8 displays equilibrium profiles with correlated $H^3/Q_r$ and $T$ values chosen to better represent: (i) the

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Fig. 6. Profiles across the well-resolved, convex upward portions of the subaqueous deltas/clinoforms offshore of the Rhone, Po, Waipu, Ganges-Brahmaputra and Eel Rivers. The depths at which the slope of each profile reaches 0.01 are given in Table 1.
Rhone and Po, (ii) the Waiapu and Ganges, and (iii) the Eel.

5. Discussion and conclusions

The underlying premise of this paper is that highly turbid hyperpycnal layers, nourished by river supply and supported by wave-driven resuspension, move down slope across the shelf under the influence of gravity. This premise is supported by several recent studies associated with the STRATAFORM Program (Ogston et al., 2000; Traykovski et al., 2000; Wright et al., 2001, 2002; Scully et al., 2002, 2003). Previous evidence indicates that deposition from the turbid mud layers occurs due to across-shelf gradients in wave energy and/or bed slope. Based on this evidence, we have offered a simple model for an equilibrium cross sectional profile shape for shelves dominated by gravity-driven transport of mud to explain the convex

Fig. 7. (a) Depths at which bathymetric profiles reach 0.01 \( h_{0.01} \) as observed and as predicted by Eq. (9), both as a function of rms wave height \( (H)^3 \) cubed divided by riverine sediment discharge per unit length along shelf \( (Q_r) \). (b) Comparison of observed \( h_{0.01} \) to that predicted by Eq. (8).
portion of subaqueous deltas/clinoforms observed off many rivers. The two key parameters governing the shape of the profile are rms wave height and river sediment discharge per unit length along-shelf during flood events. Wave period also contributes, but tends to be correlated to wave height. Because the maximum fine sediment load is determined by the critical Richardson number, the equilibrium profile is not dependent on other sediment or bed properties. A comparison of the theory to the observed slopes and depths of subaqueous deltas and clinoforms at STRATAFORM, EuroSTRATAFORM and other sites shows a good fit at the lowest order. The model offers a first-order explanation of the delta-front/shelf profiles that exist off the mouths of the Eel, Waiapu, Po, Rhone and Ganges Rivers.

The equilibrium condition as we portray it here represents an event-averaged state whereby the supply of river sediment near the “top” of the convex portion of the profile is just balanced by gravity-driven bypassing of an equal amount of sediment at its base. The implication is, of course, that for equilibrium to prevail the ultimate sink for the river sediment must be in deep water at or beyond the base of the equilibrium portion. However, some degree of disequilibrium is more likely the norm and must give rise to morphodynamic evolution of even the convex portion of the profile. For example, where a subaqueous delta is perched upon a deeper existing shelf, the underlying shelf constrains the profile slope at depth, limiting the ability of wave supported gravity currents to bypass sediment entirely off the shelf. Accretion at the base of the delta must then occur, eventually affecting the profile gradient at shallower depths, eventually leading to vertical accretion nearer the shore and/or shoreline progradation. Future analyses will address the directions and rates of such changes.

In the analytical model that we present here, only wave agitation and river supply of sediment have been considered. However, strong along-shelf tidal, wind- and/or buoyancy-driven currents commonly accompany or exceed wave agitation in the vicinity of major river systems. Relevant examples include the East China Sea, Gulf of Bohai, Gulf of Thailand, Gulf of Papua, Amazon, Orinoco, and the Apennine rivers of the Adriatic Sea. In the near future, we intend to expand our understanding to better embrace such situations.

List of Symbols

- $A$: river basin drainage area
- $c$: mass concentration of suspended sediment within wave boundary layer
- $C$: depth-integrated mass concentration of suspended sediment within wave boundary layer
- $c_d$: bottom drag coefficient
- $g$: acceleration of gravity
- $h$: water depth
- $h_{0.01}$: water depth where bed slope equals 0.01
- $H$: rms wave height during gravity flow event
- $H_{\text{sig}}$: significant wave height
- $H_{90}$: 90th percentile rms wave height from 30-year hindcast
- $k$: wave number
- $Q_r$: supply of riverine sediment per unit distance along-shelf
- $R_i$: Richardson number
- $R_i_c$: critical Richardson number
- $s$: submerged weight of siliceous sediment relative to water
\( T \) \hspace{1cm} \text{bottom orbital peak period at depth where bed slope equals 0.01}

|\( |u| \) | \text{absolute amplitude of instantaneous velocity} |
|---|---|
|\( u_g \) | \text{downslope velocity of gravity current} |
|\( u_w \) | \text{rms near-bed wave orbital velocity} |
|\( \alpha \) | \text{sine of bed slope} |
|\( \alpha_c \) | \text{critical bed slope} |
|\( \delta \) | \text{thickness of wave boundary layer} |
|\( \theta \) | \text{bed slope in degrees} |
|\( \rho_s \) | \text{density of siliceous sediment} |
|\( \omega \) | \text{wave radian frequency} |

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