Critical Bed-Shear Stress for Cohesive Sediment Deposition under Steady Flows

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Abstract: Results of two laboratory experiments on the cohesive sediment deposition behavior are presented. Data indicate that an annular flume, either with or without the channel bed and the top ring rotating in opposite directions to minimize the secondary circulation, can be used to study the deposition behavior of cohesive sediments. Direct observations on when and where the bed is formed suggest that deposition only occurs when the local bed-shear stress ($\tau_b$) is less than a critical value. Secondary circulation in the flume, which produced upward current near the inner corner and downward current near the outer corner, did not prevent deposition near the inner corner (because of the small $\tau_b$) or promote deposition near the outer corner (because of the relatively large $\tau_b$).

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Introduction

The classical approach to the deposition of cohesive sediments (Krone 1962, 1993; Mehta 1973; Mehta and Partheniades 1975) uses a parameter, “the critical bed-shear stress for deposition ($\tau_{cd}$),” and the maximum possible sediment downward flux ($w_sC$) above the water–sediment interface to represent the deposition rate, $D$. Here, $w_s=$settling velocity and $C=$near bed suspended sediment concentration (SSC). The above approach is based on the definition that deposition is a process by which a sediment particle or floc comes to the bed, and most importantly, sticks to the bed (Krone 1993). Krone (1962) suggested the widely used formulation for $D$ as follows:

$$D = 0, \text{ when } \tau_b \geq \tau_{cd} \quad (1a)$$

$$D = -pw_sC, \text{ when } \tau_b < \tau_{cd} \quad (1b)$$

where $\tau_b=$local bed-shear stress; and $p=1-\tau_b/\tau_{cd}=$probability for deposition.

For any numerical modeling effort, parameters for the bottom boundary condition, i.e., the critical bed-shear stress for erosion to start ($\tau_{ce}$), the erosion rate ($E$), the critical bed-shear stress allowing sediment to deposit ($\tau_{cd}$), and the deposition rate are essential.

The existence of $\tau_{ce}$ is widely accepted, but the existence of $\tau_{ce}$ is still being debated.

At a Chesapeake Bay site, Sanford and Halka (1993) found that SSC in a water column began decreasing soon after $\tau_b$ started to decrease (i.e., in the deceleration phases), both before low slacks as well as before high slacks. This suggested that deposition was not affected by different water depths, which is a parameter to calculate the carrying capacity (Toorman 2002; Winterwerp 2002). Because $\tau_b$ is still quite large early in the tidal deceleration phases and the assumption of a constant $\tau_{cd}$ for the sediment bed, Sanford and Halka (1993) argued that deposition must happen at all times. Because deposition happens while $\tau_b$ is still large, they concluded that $\tau_{cd}$ (which is small) must be nonexistent. Their conclusion, however, was based on (1) a constant $\tau_{cd}$ for the sediment bed; and (2) the SSC changes measured at and above 25 cm from the bed, not the total SSC from the entire water column. In other words, their observation only firmly indicated that sediment downward flux (at 25 cm above the bed) happens at all times, but not necessary that deposition also happens at all times. Net sediment downward flux observed in the water column, even at an elevation close to the bed, does not necessarily imply deposition. The assumption of a constant $\tau_{ce}$ is also not true based on evidence from others (e.g., Sanford and Maa 2001; Maa and Kim 2002) found at later times.

Maa and Kim (2002) carried out in situ erosion experiments at the York River (a tributary of the Chesapeake Bay), and they found that (1) $\tau_{ce}(z)$ increases with sediment depth, $z$, where $z=0$ at the water–sediment interface and increases downward; and (2) erosion is a relatively rapid process: $E$ approaches zero for a constant $\tau_b$ after about 20 min in most estuarial environments. Because of the slow-changing tidal force and the gradient of $\tau_{ce}(z)$ at the water–sediment interface, they concluded that erosion in most estuaries, where $\tau_{ce}(z)$ increases with depth, could only occur when the tide is in acceleration phases (i.e., when $\tau_b$ increases with time). Their statements were also supported by their tripod observations of SSC in the water column at two observing elevations where the SSC starts to increase immediately after the tidal acceleration phases and starts to decrease soon after $\tau_b$ starts.
Experimental Setup

Two annular flumes were used in this study: (1) The Virginia Institute of Marine Science (VIMS) laboratory carousel (Maa et al., 1995) which drives the contained fluid using the top ring only; and (2) a laboratory carousel built at Chonbuk National University, South Korea, which allows the top ring and channel bed to rotate in opposite directions to reduce secondary circulation. These two flumes have exactly the same dimensions: channel width=15 cm, channel depth=10 cm, inner wall radius=1.0 m, and outer wall radius=1.15 m. Although the flume at Chonbuk University allows the top ring and channel bed to rotate in opposite directions, the channel bed was kept stationary in order to provide the same function as the VIMS laboratory carousel.

Experiments Conducted at Chonbuk University

Experiments in the Chonbuk flume mimicking the deposition experiments carried out by Mehta (1973), checked whether a relatively strong secondary circulation will affect the experimental results. In Mehta’s experiments, the top ring and the channel bed rotated in opposite directions in such a manner that the secondary circulation was minimized, if not totally eliminated.

A commercially available kaolinite [the same brand as that used by Mehta (1973)] with a median diameter of $d_{50} = 4 \mu m$ was used in this study. The kaolinite was first mixed with tap water and soaked for 2 days before use. Sediment slurry with an initial SSC ($C_0$) of about 1 g/L was then prepared and put in the flume. At this time, the ring started rotating with a high speed to provide a large average bed-shear stress, $\tau_{bg} (=1.8 Pa)$ for keeping the sediment in suspension and for further mixing the sediment in the flume. This process was maintained for 15 min and then the ring speed (i.e., $\tau_{bg}$) was reduced to a preselected value to start the deposition test. The change of SSC in the flume was observed by withdrawing water samples from two ports on the outer wall (4 and 7 cm above the bed) at selected times. The water samples were then filtered and dried in an oven at 100°C for 24 h to obtain the dry weight for calculating the SSC. Average SSCs from these two ports were used to interpret the deposition process. The above procedures were repeated many times with the same $C_0$ but different $\tau_{bg}$.

Experiments Conducted at VIMS

The deposition test conducted at the VIMS laboratory carousel used a time series of selected constant $\tau_{bg}$ with different durations to find $\tau_{cd}$. This approach is similar to that used to identify $\tau_{cr}$ (Fukuda 1978; Parchure and Mehta 1985; Sanford and Maa 2001), and thus, should have a similar results for identifying $\tau_{cd}$. Sediment from Mai-Po, Hong Kong, with a deflocculated $d_{50}$ around 2 $\mu m$, was used. Results of clay mineral identification indicated roughly 51% of kaolinite, 25% of smectite, and 24% of muscovite. The initial concentration was arbitrarily prepared as $C_0=3.5$ g/L and the salinity was set at 30 parts per trillion (ppt) using sea salt. The Mai-Po mud was placed in the flume and then eroded by applying a high $\tau_{bg}$ (0.85 Pa) for 9 h in order to completely erode the mud. During this period, the SSC increased quickly in the beginning but then slowed down significantly. However, even at the end of this 9-h period, the SSC still increased slightly, and there was a small portion of mud (about 0.8 cm) near the inner wall that was not yet eroded. This phenomenon can be explained by the small $\tau_{bg}$ near the inner wall even with a high $\tau_{bg}$ (see the next section) and the turbulent flow in this flume (Parchure and Mehta 1985).

The change of SSC was monitored continuously by using an optical backscatter sensor (OBS) (Downing et al. 1981) mounted at the middle elevation of the inner wall. OBS outputs were calibrated with the SSC’s measured from water samples withdrawn from three ports at different elevations on the outer wall at selected times. SSC measurements from these multiple water samples confirmed that the water was well mixed.

Bed-Shear Stress Distribution

Local bed-shear stress ($\tau_b$) is the vector sum of two components: a major component caused by the tangential flow and a minor component caused by the secondary flow. In the flume, $\tau_b$ varies in the radial direction (see Fig. 1) with small $\tau_b$ near both walls because of the wall effect (Maa 1993), and actually $\tau_b=0$ at $R=1.0$ and 1.15 m. The area of small $\tau_b$ is much larger near the inner wall when compared with that near the outer wall. These two areas (where $\tau_b<\tau_{cd}$) could provide areas for sediment to deposit first. Because the flow is axially symmetrical, the area of
deposition can be represented by the length of deposition, \( d \) (Fig. 1). When \( \tau_{ba} \) is large (e.g., \( \tau_{b1} \) in Fig. 1), \( d \) is small. On the other hand, when \( \tau_{ba} \) is small, \( d \) increases quickly (see \( \tau_{b3} \) in Fig. 1).

The pattern of \( \tau_b \) distribution (Fig. 1) was obtained based on a numerical simulation of flow fields in the VIMS Sea Carousel (Maa 1993) using a simple mixing-length model for turbulent eddy viscosity. Although the simulated \( \tau_{ba} \) and the distribution have been confirmed by a series of laboratory experiments (Maa et al., 1995), verification of the details near the corners is not available. Thus, the predicted \( \tau_b \) distribution near corners may be less reliable because of the use of the simple mixing-length model. Nevertheless, \( \tau_{ba} \) were used in experiment design, and later, to interpret the experimental results.

In order to ignore the sidewall effect, a minimum width/depth ratio of 5 is necessary and a ratio of 10 is preferred (Chow 1959). Unfortunately, most of the studies on cohesive sediment dynamics were carried out at small flumes without meeting this requirement, yet the sidewall effect was ignored in the data analysis.

**Observation from Bottom**

The transparent Plexiglass bottom of the VIMS laboratory carousel permitted direct observation of sediment deposition. Thus, two approaches to interpret the experimental results were available: (1) using the change of total SSC; and (2) observing the change of deposition length directly from the flume bottom. Note here that the measured SSC in this study accurately represents the total SSC because of the relatively strong secondary circulation for mixing, even for low bed-shear stresses.

**Experimental Results from the Chonbuk Flume**

The results of the Chonbuk flume experiments indicated that an equilibrium state (when the SSC does not change anymore) was achieved after a few hours for large \( \tau_{ba} \) (i.e., \( \tau_{ba} \geq 0.20 \text{ Pa} \)) and the larger the \( \tau_{ba} \), the higher the ratio of \( C_{eq}/C_0 \), where \( C_{eq} \) = SSC at the equilibrium states (Fig. 2). When \( \tau_{ba} < 0.165 \text{ Pa} \), it appears that all the suspended sediments deposited. The Chonbuk experiments differ from those of Mehta (1973) in that the Chonbuk flume has a relatively strong secondary circulation, and thus, a reasonably uniform SSC in the water column. In contrast, the flume of Mehta (1973) had negligible secondary circulation, and thus, a near-bottom thin fluffy layer (i.e., stirred layer) (Mehta 1991) could exist when there is a net downward flux of sediment. The Chonbuk flume also had a relatively large flume radius and width/high ratio and the experiments used tap water instead of distilled water. As such, the kaolinite in the study of Mehta may have been less aggregated. Despite these differences, the measured average bed-shear stress for all sediments to deposit, \( \tau_{ba, min} \) (slightly less than 0.16 Pa for this study) is only slightly different from that (about 0.18 Pa) obtained by Mehta (1973). It is hard to determine precisely the value of \( \tau_{ba, min} \) because the experiment durations were not sufficiently long (Fig. 2). Nevertheless, the small difference in \( \tau_{ba, min} \) demonstrates that secondary circulation has a negligible effect on the deposition process, and the bed-shear stress is the control parameter for deposition.

**Experimental Results from the VIMS Laboratory Carousel**

After erosion, the first low \( \tau_{ba} \) (0.65 Pa) only reduced the SSC slightly [Figs. 3(a and b)], such that the region where \( \tau_b \approx \tau_c \) near the inner wall increased only slightly, as shown by the slight increase of deposition length, \( d \) [Fig. 3(c)]. For the second lower \( \tau_{ba} \) (0.35 Pa), the SSC first showed a sharp decrease, then approached an equilibrium state, and \( d \) also increased quickly to 1.6 cm and then slowly increased to 1.9 cm at the end of this \( \tau_{ba} \). When \( \tau_{ba} \) was further decreased to 0.12 Pa, the SSC again decreased sharply at first, but the time required to approach the equilibrium state was more than 2 h, and \( d \) also increased quickly to 2.2 cm but did not grow significantly for the remaining duration of this \( \tau_{ba} \). Similar responses were observed for the next two lower \( \tau_{ba} \) (0.096 and 0.067 Pa). There was an initial small decrease of SSC and a slight increase in \( d \), but which remained constant thereafter.

When \( \tau_{ba} \) was further decreased to 0.042 Pa, the SSC decreased quickly and apparently reached a SSC < 100 mg/L after 2 h. The deposition length, \( d \), increased sharply from 3.4 cm at time=1,243 min to 7.2 cm at time=1,281 min, and then slowly
increased to 7.3 cm at time \( t = 1,515 \) min. At that time, there was little sediment in suspension (see the almost clear water on the left in Fig. 4), and thus, when \( \tau_{ba} \) was further reduced, the deposition length only increased a little, up to 7.8 cm. Because of the low SSC (i.e., little sediment to deposit) at the last \( \tau_{ba} \), the measurement of deposition length is less accurate. At this time, even the area for deposition \( (\tau_b < \tau_{cd}) \) near the outer corner is sufficiently large for visual observation, but there is no sediment for deposition.

The significant increase of deposition length when changing \( \tau_{ba} \) from 0.067 to 0.042 Pa suggests that the latter \( \tau_{ba} \) (0.042 Pa) is close to \( \tau_{cd} \), and thus, a \( \tau_{cd} \approx 0.04 \) Pa was assigned.

**Discussion and Conclusions**

**Upward Diffusive Transport in the Water Column**

Although this process always exists, the transport amount may change significantly, depending on the availability of the sediment supply. When there is a sufficient supply from the water–sediment interface (e.g., caused by erosion at tidal acceleration phases) and when the eddy diffusivity \( (e) \) is also strong \( (e \propto \text{shear velocity, } u^* \text{)} \) (Henderson 1966), the upward diffusion flux may be larger than the downward settling flux. Thus, the SSC at a practical observation elevation (e.g., 10 cm above bed) increases with time. When there is no supply from the water–sediment interface (e.g., when erosion ceases or/and the turbulent kinetic energy is also reducing) (Winterwerp 2002), then the amount of upward transport would be small and the result is a decreasing SSC at the selected practical observation elevation.

**Role of Local Bed-Shear Stress, \( \tau_b \)**

Because \( \tau_b = 0 \) at the inner corner \( (R = 1.0 \text{ m}) \) and a small area for \( \tau_b < \tau_{cd} \) even when \( \tau_{ba} \) is large \( (\text{e.g., } \tau_{ba} = 0.85 \text{ Pa}) \), a small \( d \) at the inner corner was observed. Although \( \tau_b = 0 \) at the outer corner \( (R = 1.15 \text{ m}) \), the low \( \tau_b \) area was too small for visual identification during most \( \tau_{ba} \). When this area was sufficiently large for visual observation (i.e., when \( \tau_{ba} < 0.04 \) Pa), there was little sediment in the water column for deposition.

The deposition length, \( d \), is determined by \( \tau_{cd} \) and the \( \tau_b \) distribution, see Fig. 1. When \( \tau_{ba} \) was reduced to 0.042 Pa, \( d \) increased significantly because \( \tau_{cd} \) was close to 0.042 Pa. Thus, a sharp increase of \( d \) implies that \( \tau_{ba} \approx \tau_{cd} \).

Notice that deposition happened at all times near the inner sidewall where \( \tau_b < \tau_{cd} \). This explains the continuous decrease of SSC even when \( d \) remains the same (e.g., see Fig. 3, for 870 min < time < 1,020 min, or 1,050 min < time < 1,240 min). During this period, the decrease of SSC contributes to the build up of bed thickness in the deposition zone, not to the growth of \( d \). The amount of deposition, however, depends on SSC. For low SSC, this amount would be insignificant and hard to observe. If the accumulation of sediment near the inner corner is thick, the hydrodynamic and \( \tau_b \) distribution will be affected. Nevertheless, \( \tau_b \) would be still small near the corner, but the area for \( \tau_b < \tau_{cd} \) would be reduced.

**Role of Convective Transport**

This kind of transport \( (wC, \text{ where } w \text{ is the vertical component of water velocity}) \) only contributes to the change of SSC in the water column, and its role in deposition appears to be unimportant. This is because the cohesive binding force between cohesive sediment particles/flocs (or between sediment and the Plexiglas bed) is the only force to keep the sediment particle/floc at the bed. If \( \tau_b \) is larger than this binding force, then the particle/floc that comes to the bed cannot stick to the bed.

Secondary circulation in the flume produced a downward flow near the outer corner and an upward flow near the inner corner, but it did not prevent deposition at the inner corner, nor did it cause deposition near the outer corner. This demonstrates that the secondary circulation has little effect on deposition. Its role on transport, however, is obvious. For example, this convective transport brings sediment to the low \( \tau_b \) area near the inner corner for deposition.

**Role of SSC**

When the SSC in the flume is low, the amount of sediment that is available for the possible deposition would be also small, and thus, the change of deposition length may not be noticeable. In other words, the following two conditions are required to produce a noticeable change of deposition length: (1) A relatively small \( \tau_{ba} \) and a relatively large \( \Delta \tau_{ba} \) and (2) a sufficiently large SSC in the carrying water for deposition.

**Buffer Layer**

When (1) there is a net sediment downward flux toward the water–sediment interface; (2) no secondary circulation to mix it up; and (3) \( \tau_b > \tau_{cd} \) the sediment mass may be accumulated in a small zone right above the water–sediment interface and a buffer layer may be developed. The sediment in this layer is still moving with the benthic flow, and it is not stationary. This buffer layer is similar to the “stirred layer” discussed by Mehta (1991). With the first two conditions given above but \( \tau_b < \tau_{cd} \), the sediment moved down to the bed will stick to the bed and start to consolidate immediately.

In general, this buffer/stirred layer is thin, on the order of millimeters to centimeters. If the amount of net sediment downward flux is sufficiently large, this layer may become a fluid mud (roughly with SSC more than 10 g/L) layer with a relatively large thickness. When this happens, the hydrodynamic condition for the
entire water–fluid mud system would be different because of the non-Newtonian properties of fluid mud, and thus, is beyond the scope of this study.

It is a difficult, if not an impossible, task to prove the existence of the buffer layer. The existence of this layer in an environment without secondary flow, however, is expected because there is a time window in the tidal deceleration phases within which the downward sediment flux continues to bring sediment to this layer, but not allowed to deposit because $\tau_b > \tau_{cd}$.

The following are concluded from this preliminary laboratory study:
1. Deposition only happens when the local bed-shear stress ($\tau_b$) is less than a critical value, $\tau_{cd}$, which is one of the cohesive sediment properties.
2. The convective transport in the water column has little effect on deposition.

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**Notation**

The following symbols are used in this technical note:

- $C$ = suspended sediment concentration;
- $C_0$ = initial suspended sediment concentration;
- $C_{eq}$ = SSC at the equilibrium (or steady) state during a deposition test;
- $d$ = deposition length;
- $D$ = deposition rate;
- $E$ = erosion rate;
- $p$ = probability for sediment to deposit;
- $R$ = radial coordinate of the annular flume;
- $\tau_b$ = local bed-shear stress;
- $\tau_{b1} - \tau_{b3}$ = local bed-shear stress distributions;
- $\tau_{ba}$ = spatial-averaged bed-shear stress;
- $\tau_{ba1} - \tau_{ba3}$ = average bed shear bed-shear stress for $\tau_{b1}$, $\tau_{b2}$, and $\tau_{b3}$, respectively;
- $\tau_{be,min}$ = minimum $\tau_{ba}$ that will not be able to maintain a constant $C_{eq}$;
- $\tau_{cd}$ = critical bed-shear stress allowing cohesive sediment to deposit;
- $\tau_{ce}$ = critical bed-shear stress for cohesive sediment erosion to start;
- $w$ = vertical component of water velocity;
- $w_s$ = settling velocity of a sediment floc;
- $z$ = vertical coordinate, $z=0$ at the water sediment interface, and increases downward; and

$$\Delta \tau_{ba} = \text{difference in two consecutive average bed-shear stress.}$$

**References**


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